EFFICIENT PARAMETRIC AMPLIFICATION IN DOUBLE Λ SYSTEMS IN THE ABSENCE OF TWO-PHOTON MAXIMAL COHERENCE

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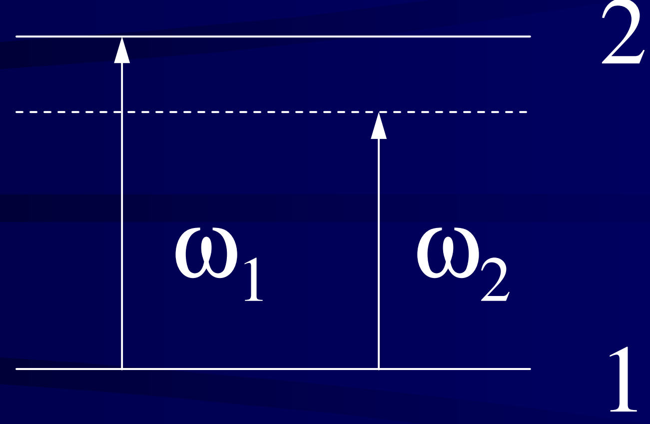
Two-level system: pump and probe



- If self-focusing and diffraction balanced, Gaussian pump propagates as a spatial soliton
- If pump-induced cross focusing of probe balanced by diffraction, the weak probe propagates as if it is a spatial soliton
- If pump is sufficiently intense, radiation generated at FWM frequency
- Parametric amplification between probe and FWM via the pump
- Process occurs over many diffraction lengths: EIPM important

Self-focusing

Nonlinear refractive index

$$n = n_0 + n_2 I$$

Thus

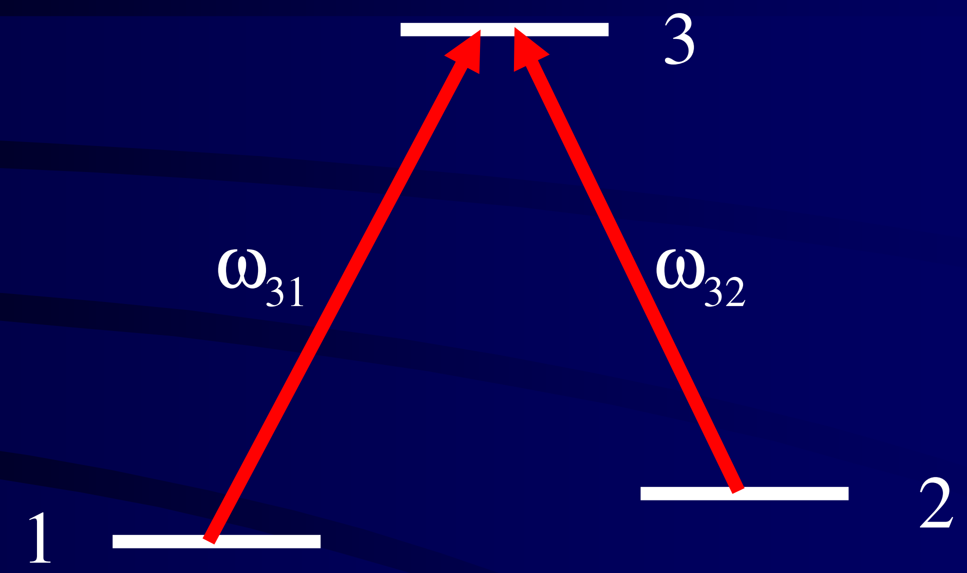
$$n_2 \Rightarrow dn/dI$$

Focusing obtained when

$$dn/dI > 0$$

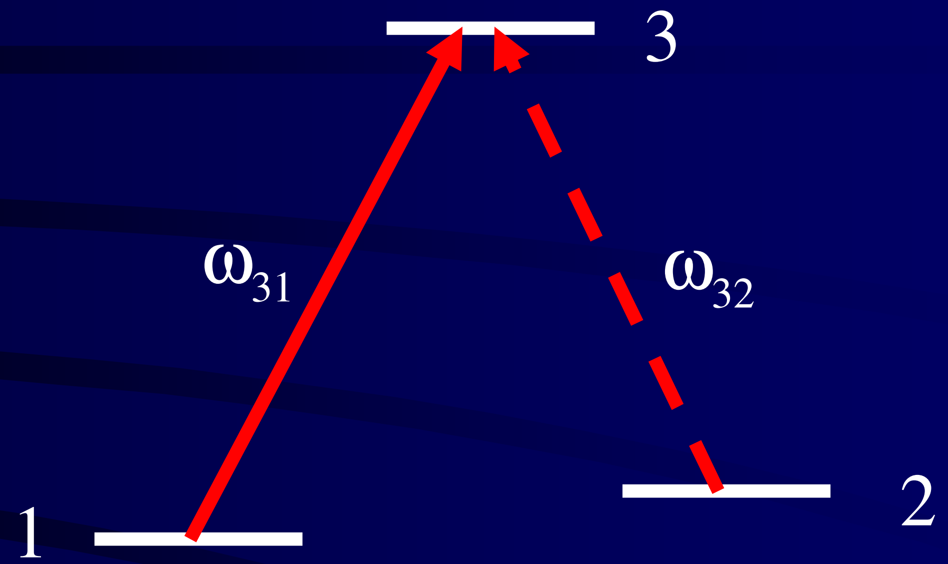
Self-focusing obtained when laser is detuned to the blue!

Coherent Population Trapping (CPT)



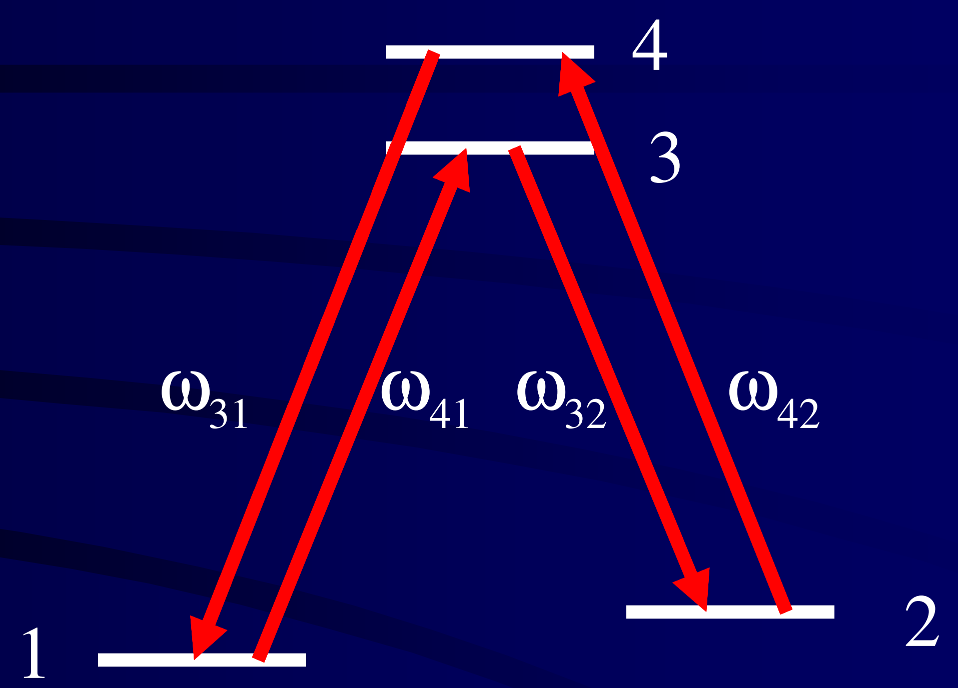
- Fields are equally intense
- Population trapped in lower levels
- Two-photon coherence is maximal: $|\rho_{21}|^2 = \rho_{11}\rho_{22} = \frac{1}{4}$
- Zero absorption

Electromagnetically Induced Transparency (EIT)



- Pump and probe fields
- Population optically pumped into state |2>
- Two-photon coherence is small

Double Λ System



- Several possible configurations
- Highly efficient FWM when CPT occurs (Harris)

Model

- Three or four beams with Gaussian transverse intensity profile (GTIP)
- Beams copropagate
- Assume steady-state
- Compare results with plane-wave beams
- Cases studied:
    - CPT with maximal coherence , either initially or on propagation
    - Four identical beams with 0 and π phase
    - Three strong fields
    - Two strong fields
    - Incoherent pumping from state 2 to 4 (not shown here)
    - Raman detuning (not shown here)

Maxwell-Bloch Equations

$$\frac{\partial}{\partial z}V'_{ij} = \frac{i}{4L_D}\nabla_T^2 V'_{ij} + \frac{i}{L_{ij}}\rho'_{ij}$$

$$\nabla_T^2 = \partial^2/\partial\xi^2 + (1/\xi)\partial/\partial\xi + (1/\xi^2)\partial^2/\partial\theta^2$$

Rabi frequency for $\lvert j\rangle \to \lvert i\rangle$ transition	V'_{ij}
Density matrix element	ρ'_{ij}
Diffraction length	L_D
Interaction length	L_{ij}
Direction of propagation	z
Transverse radial coordinate	ξ

Bloch Equations

$$\dot{\rho}_{11} = i(V_{13}\rho'_{31} + V_{14}\rho'_{41} - V_{31}\rho'_{13} - V_{41}\rho'_{14}) - \gamma_{12}\rho_{11} + \gamma_{21}\rho_{22} + \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44},$$

$$\dot{\rho}_{22} = i(V_{23}\rho'_{32} + V_{24}\rho'_{42} - V_{32}\rho'_{23} - V_{42}\rho'_{24}) + \gamma_{12}\rho_{11} - \gamma_{21}\rho_{22} + \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44},$$

$$\dot{\rho}_{33} = i(V_{31}\rho'_{13} + V_{32}\rho'_{23} - V_{13}\rho'_{31} - V_{23}\rho'_{32}) - (\gamma_{31} + \gamma_{32})\rho_{33} + \gamma_{43}\rho_{44},$$

$$\dot{\rho}_{44} = i(V_{41}\rho'_{14} + V_{42}\rho'_{24} - V_{14}\rho'_{41} - V_{24}\rho'_{42}) - (\gamma_{41} + \gamma_{42} + \gamma_{43})\rho_{44} - r_{24}(\rho_{44} - \rho_{22}),$$

$$\dot{\rho}'_{21} = i(V_{23}\rho'_{31} + aV_{24}\rho'_{41} - V_{31}\rho'_{23} - aV_{41}\rho'_{24}) - (\Gamma_{21} + i\Delta_{21})\rho'_{21},$$

$$\dot{\rho}'_{31} = i(V_{31}\rho_{11} + V_{32}\rho'_{21} - V_{31}\rho_{33} - V_{41}\rho'_{34}) - (\Gamma_{31} + i\Delta_{31})\rho'_{31},$$

$$\dot{\rho}'_{32} = i(V_{32}\rho_{22} + V_{31}\rho'_{12} - V_{32}\rho_{33} - a^{*}V_{42}\rho'_{34}) - (\Gamma_{32} + i\Delta_{32})\rho'_{32},$$

$$\dot{\rho}'_{41} = i(V_{41}\rho_{11} + a^{*}V_{42}\rho'_{21} - V_{31}\rho'_{43} - V_{41}\rho_{44}) - (\Gamma_{41} + i\Delta_{41})\rho'_{41},$$

$$\dot{\rho}'_{42} = i(V_{42}\rho_{22} + aV_{41}\rho'_{12} - aV_{32}\rho'_{43} - V_{42}\rho_{44}) - (\Gamma_{42} + i\Delta_{42})\rho'_{42},$$

$$\dot{\rho}'_{43} = i(V_{41}\rho'_{13} + a^{*}V_{42}\rho'_{23} - V_{13}\rho'_{41} - a^{*}V_{23}\rho'_{42}) - (\Gamma_{43} + i\Delta_{43})\rho'_{43}$$

Notation

- $a = \exp(i\Phi), \quad \Phi = \varphi_{31} - \varphi_{32} + \varphi_{42} - \varphi_{41}$ is initial relative phase,
- γ_{kl} is longitudinal decay rate from state $|k\rangle \to |l\rangle$,
- γ_i is total decay rate from state $|i\rangle$,
- $\Gamma_{kl} = 0.5(\gamma_k + \gamma_l) + \Gamma^*_{kl} + r_{24}\delta_{k4}\delta_{l2}$ is transverse decay rate,
- Γ^*_{kl} is rate of phase-changing collisions,
- r_{24} is rate of incoherent pumping from state $|2\rangle \to |4\rangle$,
- $\Delta_{ij} = \omega'_{ij} - \omega_{ij}$ is one-photon detuning from resonance; $i = (3,4), \ j = (1,2)$,
- $\rho'_{ij} = \rho_{ij} \exp[-i(\Delta_{ij}t + k_{ij}z - \varphi_{ij})]$,
- $\rho'_{21} = \rho_{21} \exp\{-i[(\Delta_{31} - \Delta_{32})t + (k_{31} - k_{32})z - (\varphi_{31} - \varphi_{32})]\}$,
- $\rho'_{43} = \rho_{43} \exp\{-i[(\Delta_{41} - \Delta_{31})t + (k_{41} - k_{31})z - (\varphi_{41} - \varphi_{31})]\}$.

Multiphoton Resonance Condition

$$\omega_{31} - \omega_{32} + \omega_{42} - \omega_{41} = 0$$

$$\Rightarrow \quad \Delta_{31} - \Delta_{32} = \Delta_{41} - \Delta_{42} = \Delta_{21}, \quad \text{two-photon detuning}$$

$$\text{or} \quad \Delta_{41} - \Delta_{31} = \Delta_{42} - \Delta_{32} = \Delta_{43},$$

$$\Rightarrow \quad \Delta k_0 = k_{31} - k_{32} + k_{42} - k_{41} = 0, \quad \text{initial phase mismatch}$$

Analytical Solution

$$\rho'_{ij} = \rho'^{(1)}_{ij} + \rho'^{(3)}_{ij},$$

$$\rho'_{31} = \tilde{\chi}^{(1)}_{31} V_{31} + a\tilde{\chi}^{(3)}_{31} V_{32} V_{24} V_{41},$$

$$\rho'_{32} = \tilde{\chi}^{(1)}_{32} V_{32} + a^* \tilde{\chi}^{(3)}_{32} V_{31} V_{14} V_{42},$$

$$\rho'_{41} = \tilde{\chi}^{(1)}_{41} V_{41} + a^* \tilde{\chi}^{(3)}_{41} V_{42} V_{23} V_{31},$$

$$\rho'_{42} = \tilde{\chi}^{(1)}_{31} V_{42} + a\tilde{\chi}^{(3)}_{42} V_{41} V_{13} V_{32}.$$

Real part – refraction   Imag part – absorption   FWM

Effect of phase: when Φ=0, a=1; when Φ=π, a=-1

CPT and Maximal Two-Photon Coherence

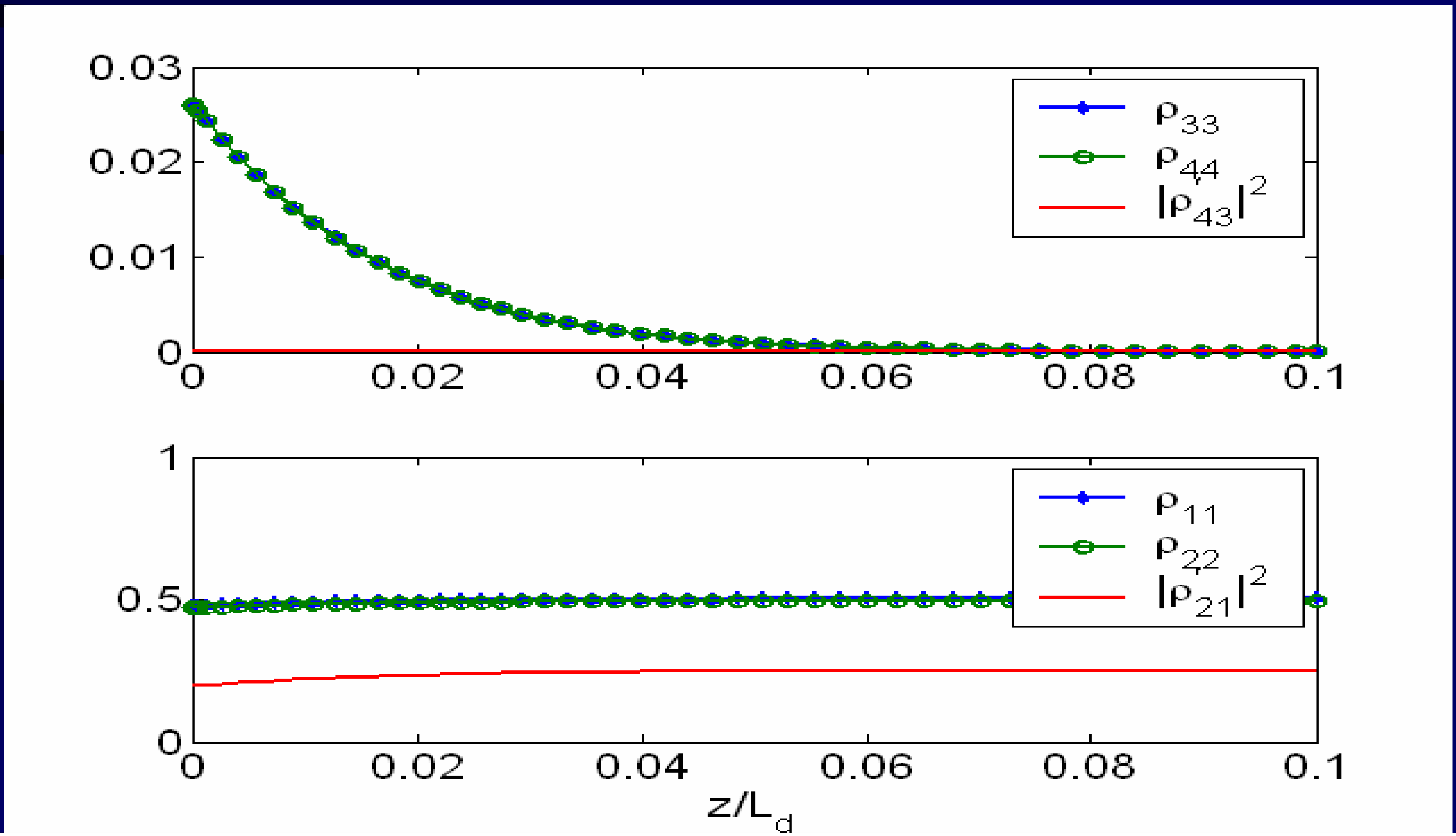
- When CPT exists, no absorption or focusing or defocusing occurs since $\chi^{(1)}=0$
- Phase-matching unimportant
- Maximum FWM occurs within a propagation distance less than diffraction length
- This is completely different from a two-level system

CPT and Maximal Two-Photon Coherence

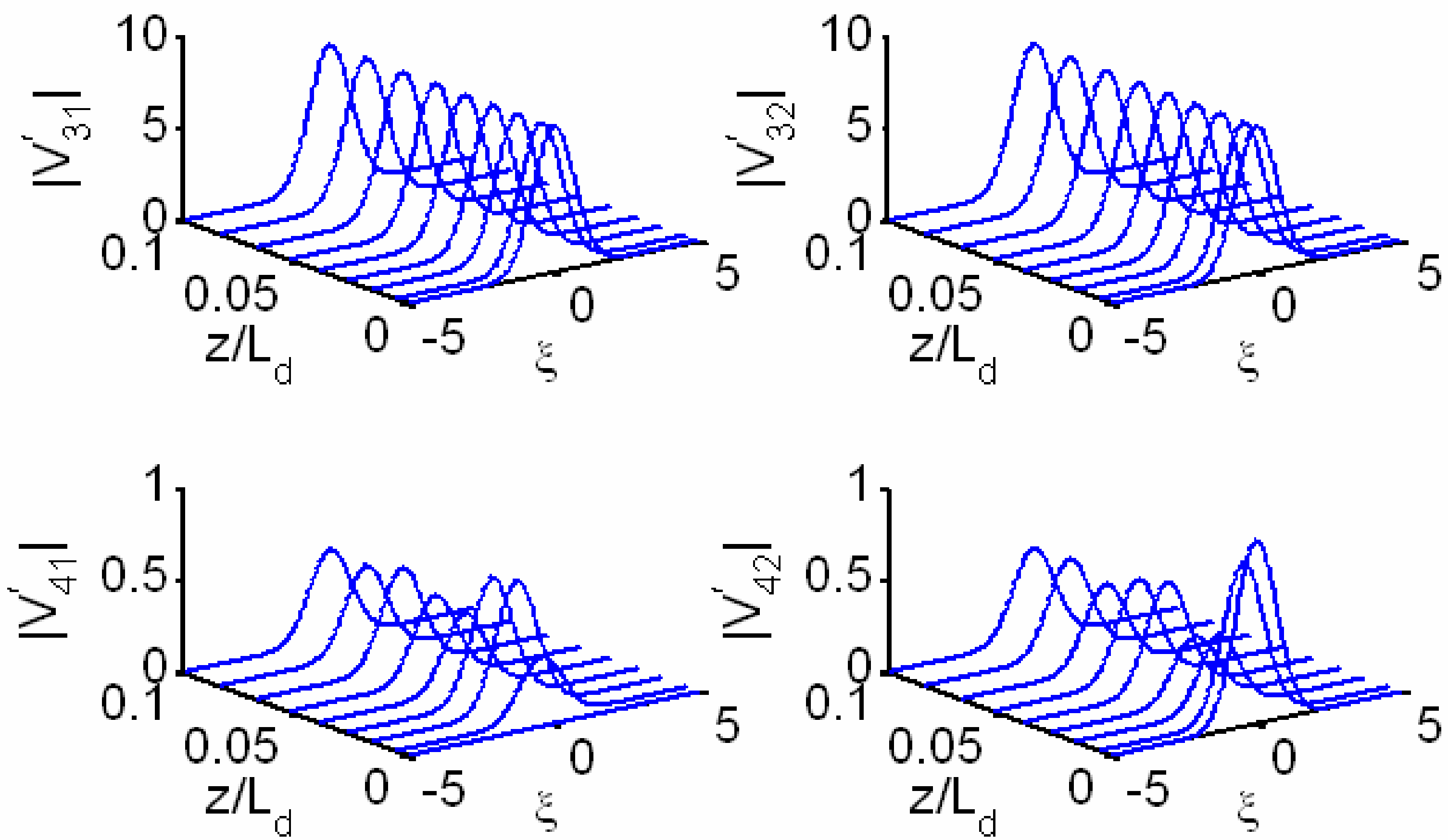
$V_{31}=8$; $V_{32}=8$; $V_{42}=1$; $V_{41}=0.001$;

$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}\pm 4$;

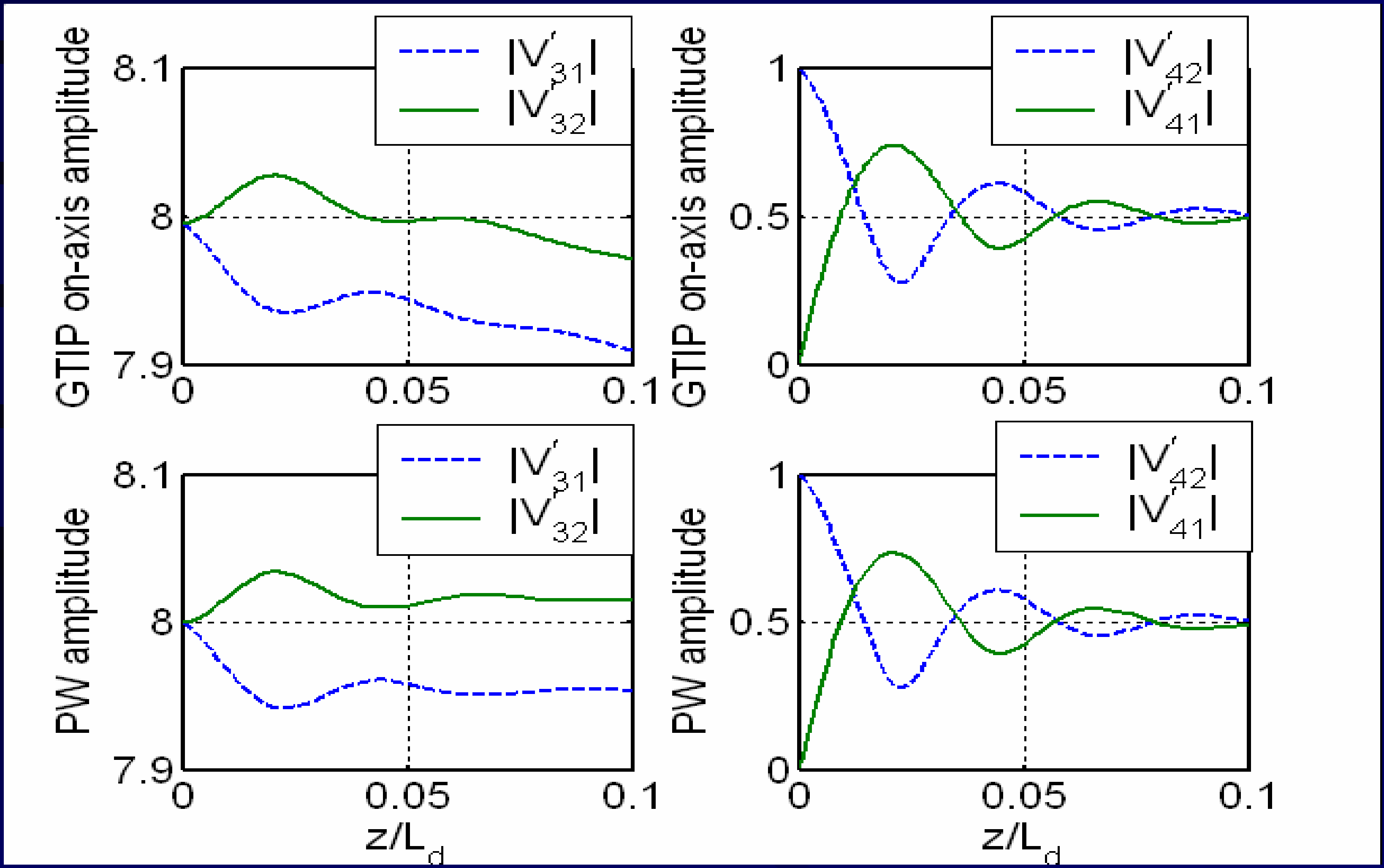
$L_{NL}/L_D=1.66\times 10^{-3}$;



Transverse Intensity Profile



Comparison: GTIP's and PW's

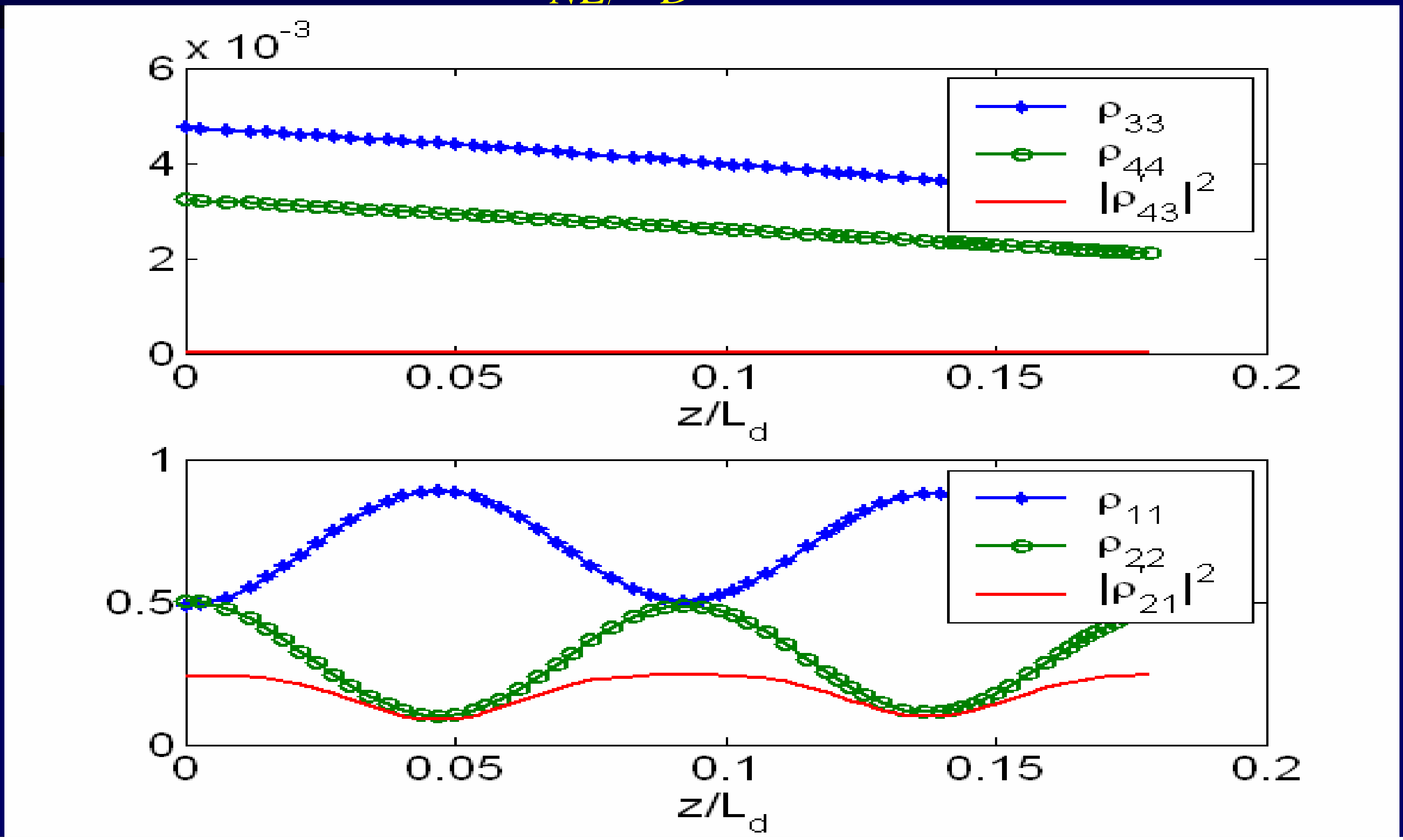


CPT and Maximal Two-Photon Coherence

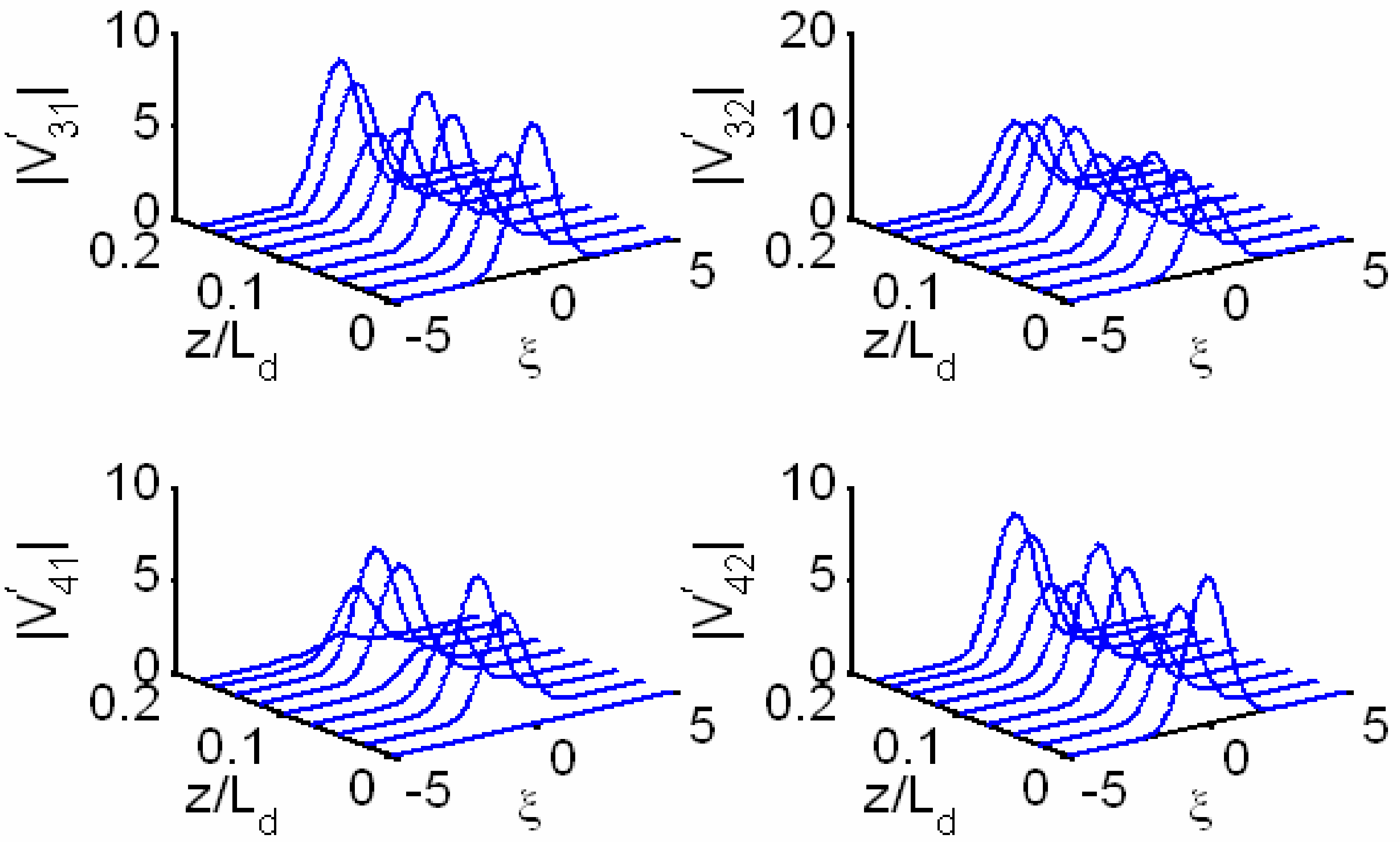
$V_{31}=8;\quad V_{32}=8;\quad V_{42}=8;\quad V_{41}=0.001;$

$\Delta_{31}=\Delta_{32}=\pm4;\ \Delta_{41}=\Delta_{42}=\pm100;$

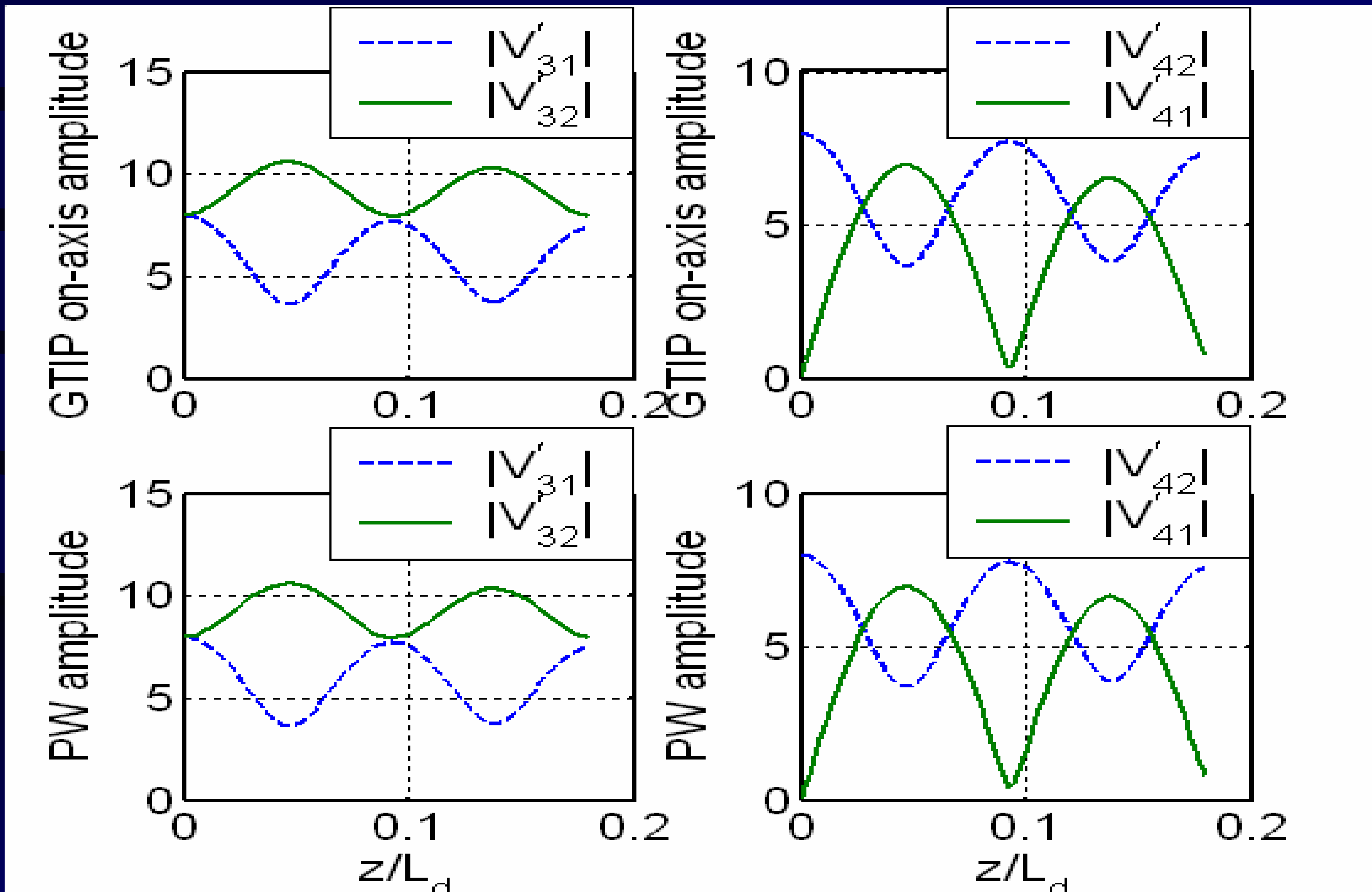
$L_{NL}/L_D=1.66\times10^{-4};$



Transverse Intensity Profile



Comparison: GTIP’s and PW’s



Onset of CPT vs. Maximum Conversion

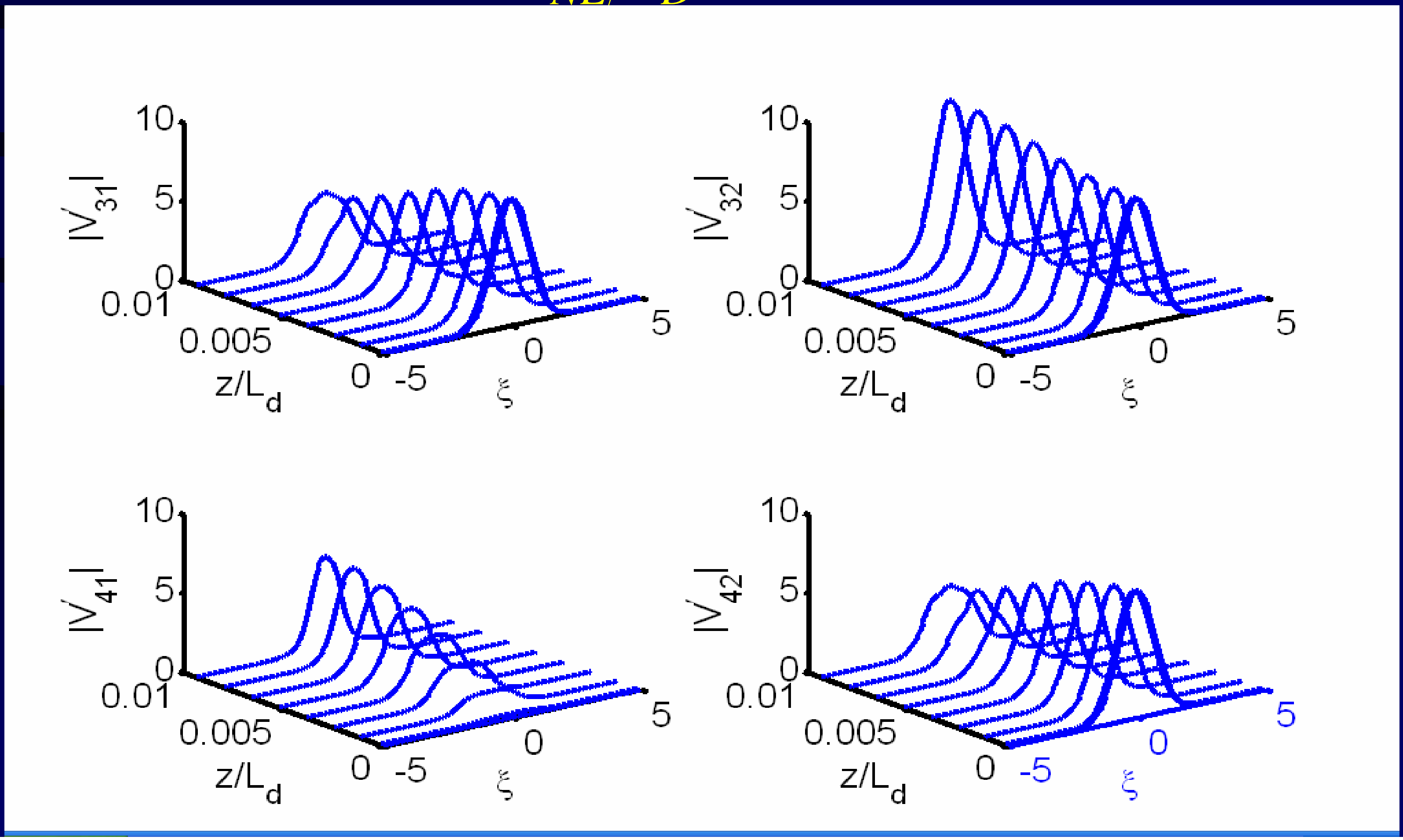
- For detuning $\Delta_{41} = \Delta_{42} = \pm 100$, CPT exists at the outset. Maximum conversion of 87% occurs at 0.047.
- For detuning $\Delta_{41} = \Delta_{42} = \pm 10$, CPT occurs at 0.1, whereas maximum conversion of 73% occurs at 0.009.
- Thus, it is possible to get efficient conversion before CPT, without focusing, defocusing or ring formation

CPT and Maximal Two-Photon Coherence

$V_{31}=8; \quad V_{32}=8; \quad V_{42}=8; \quad V_{41}=0.001;$

$\Delta_{31}=\Delta_{32}=\pm 4; \; \Delta_{41}=\Delta_{42}=\pm 10;$

$L_{NL}/L_D=1.66\times 10^{-4};$

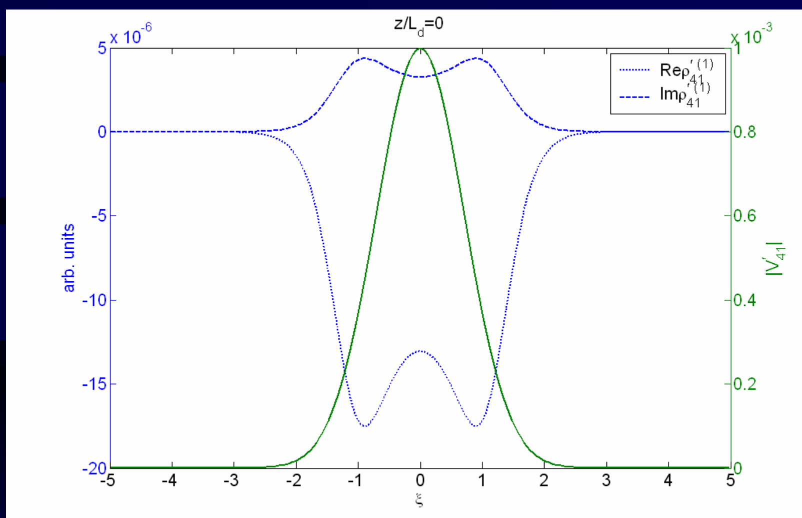


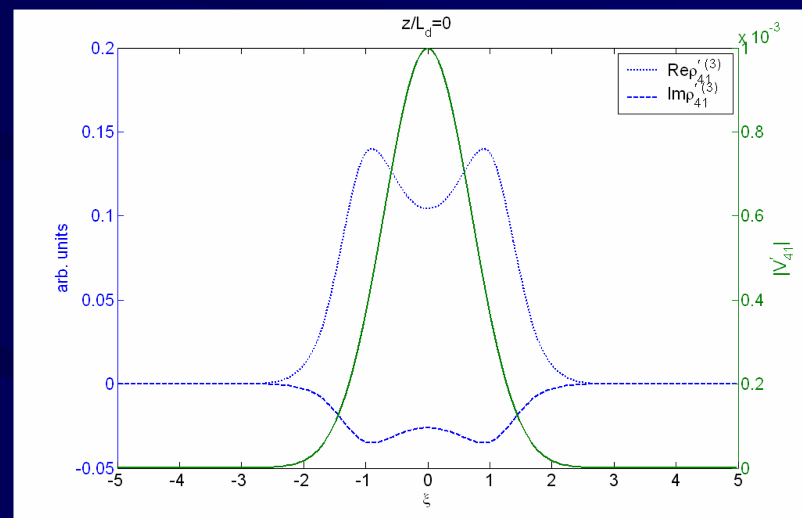
Focusing

- Focusing can occur before CPT established
- Beams blue-detuned
- Nonlinear length sufficiently long
- Maximum FWM can still occur within a short propagation distance
- Phase-matching still unimportant

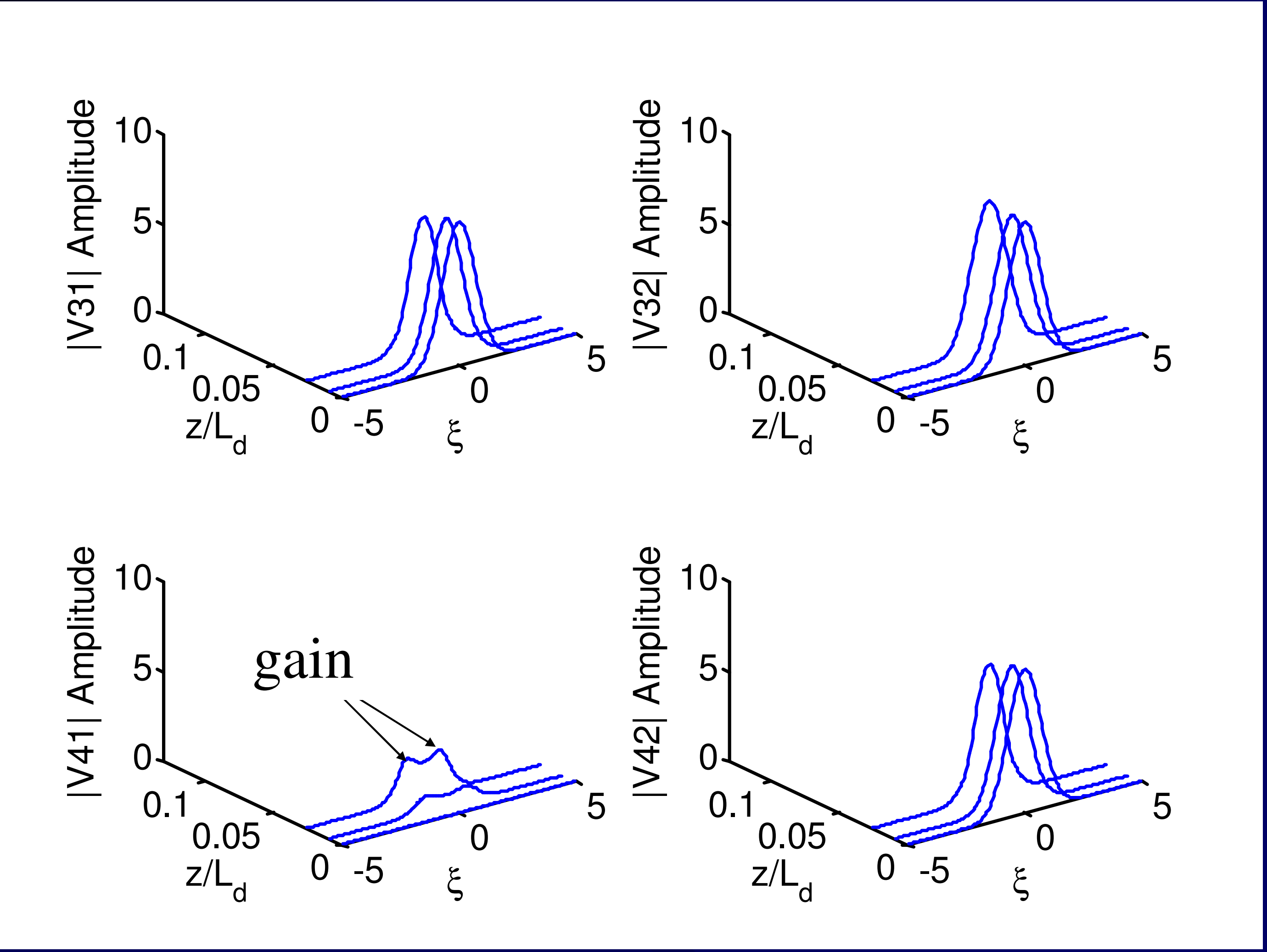
Three strong lasers

$$V_{31}=8;\quad V_{42}=8;\quad V_{32}=8;\quad V_{41}=0.001;$$
$$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}=-4;$$
$$L_L/L_D=1.52\times10^{-3};$$

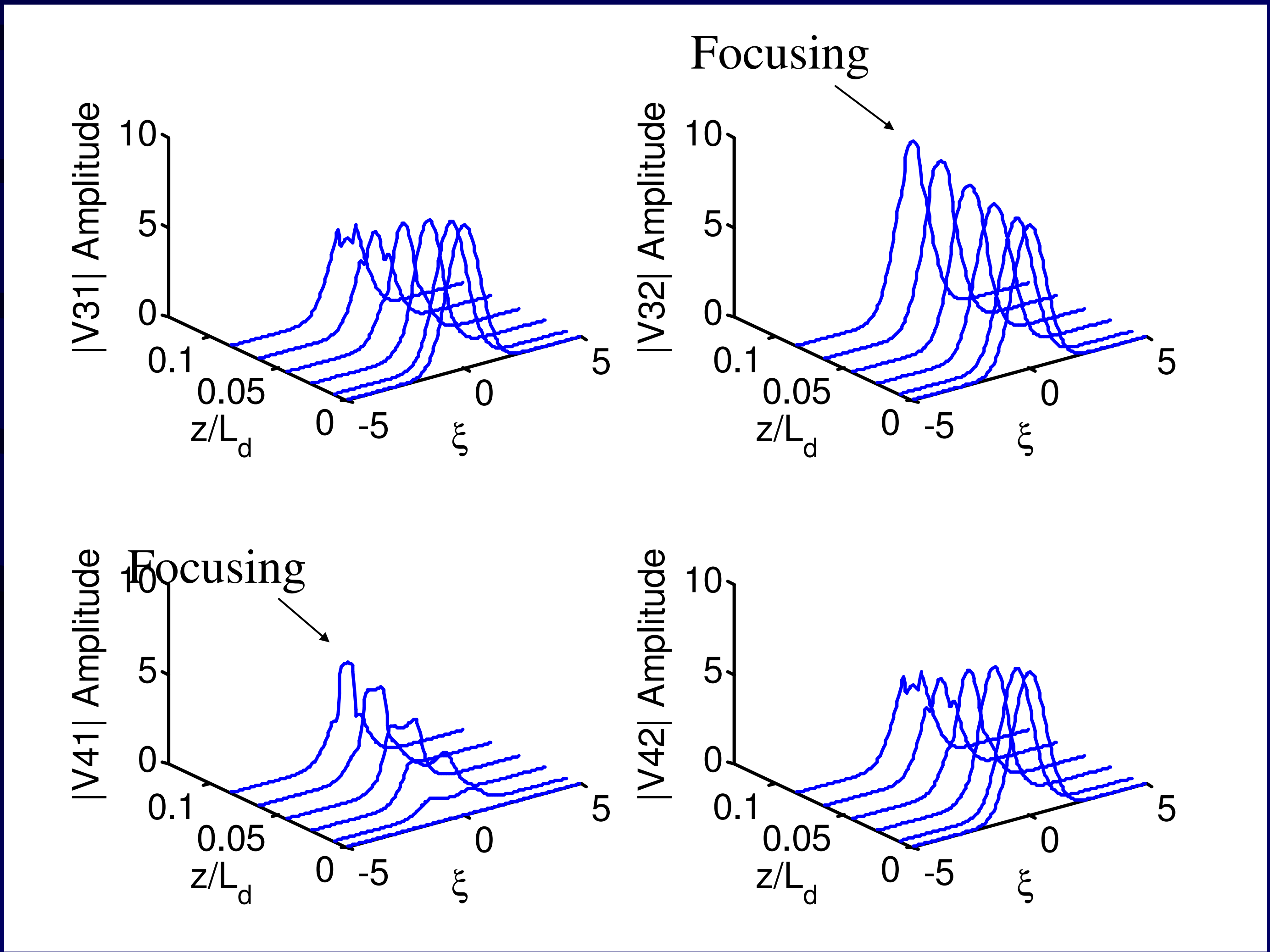




Initial gain at FWM frequency

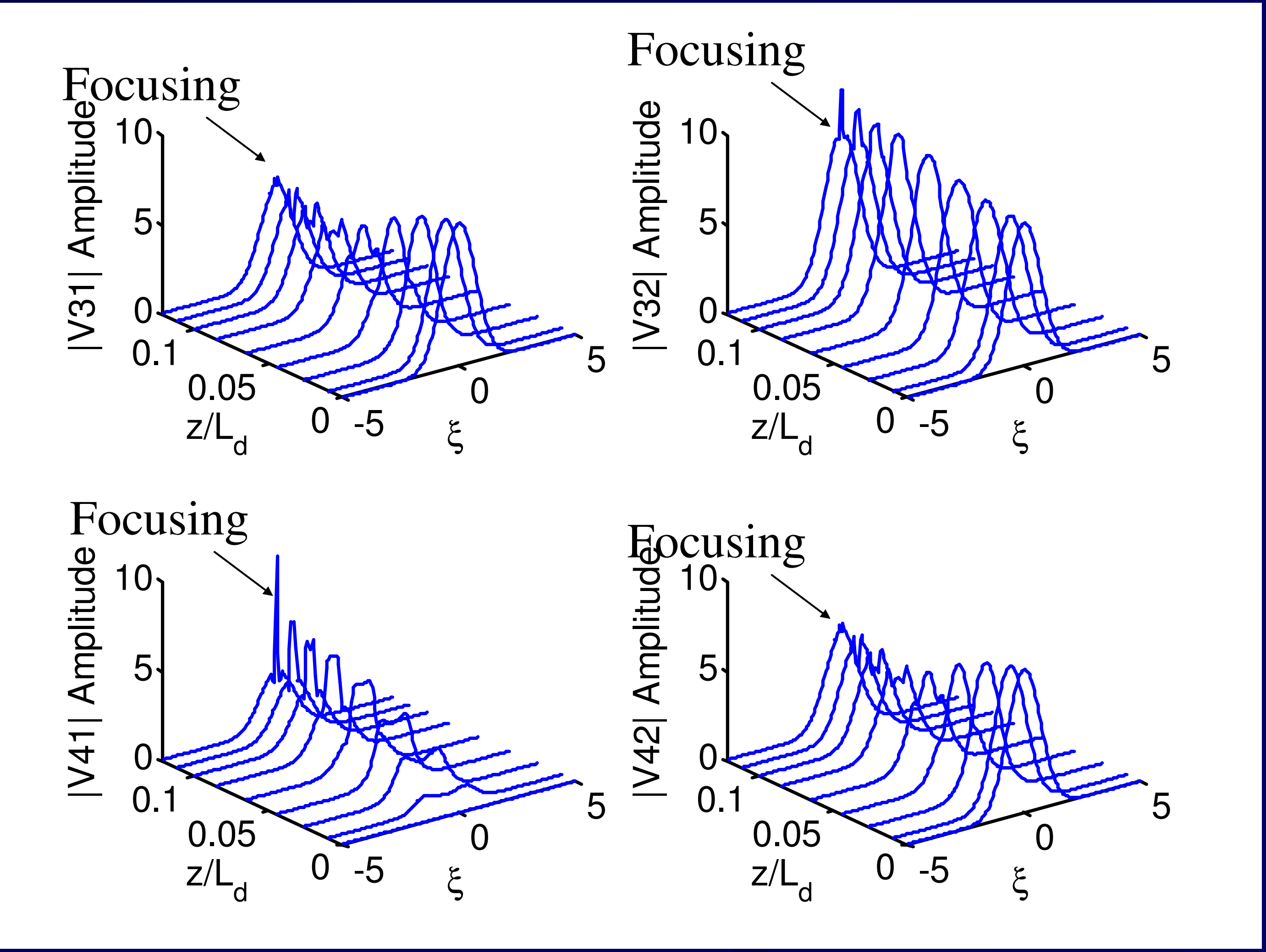


Focusing

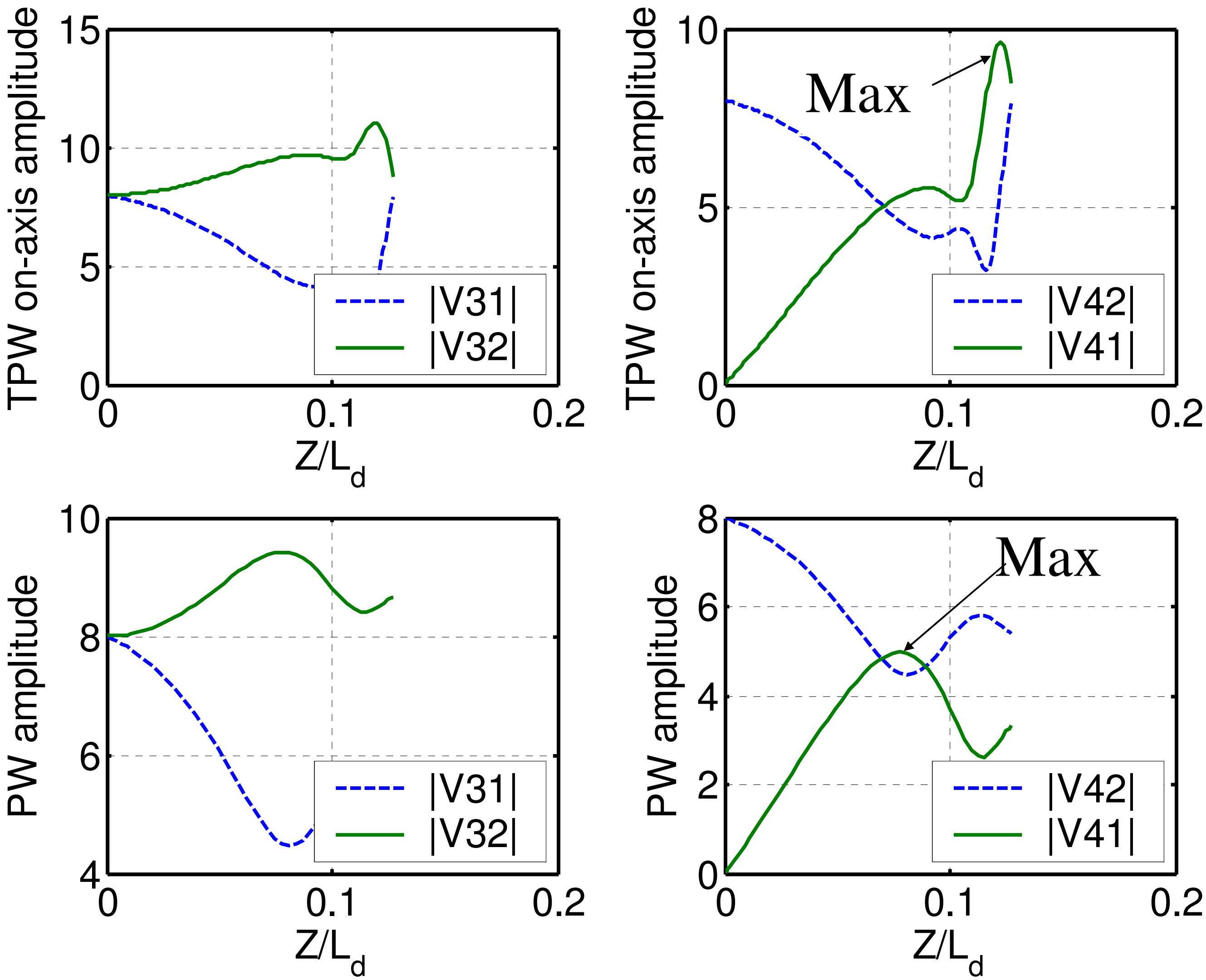


Focusing
Focusing
|V31| Amplitude
|V32| Amplitude
|V41| Amplitude
|V42| Amplitude
10
5
0
0.1
0.05
0
z/L_d
-5
0
5
ξ

Maximum focusing and conversion



Comparison: GTIP's and PW's

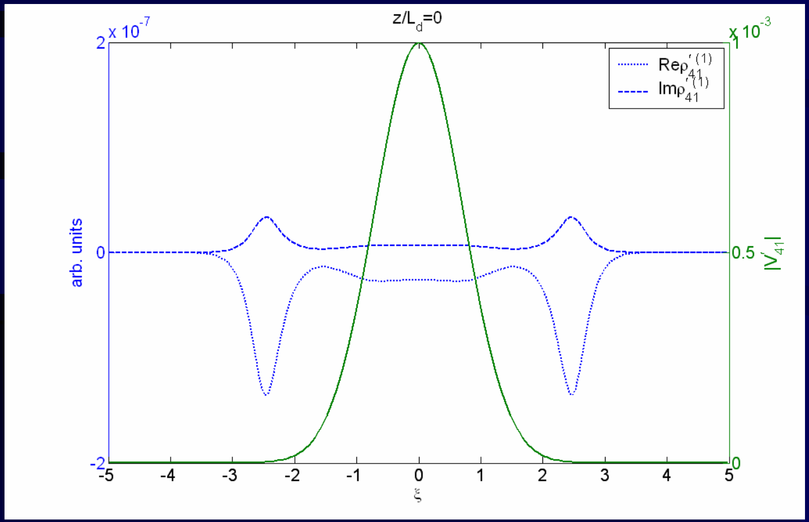


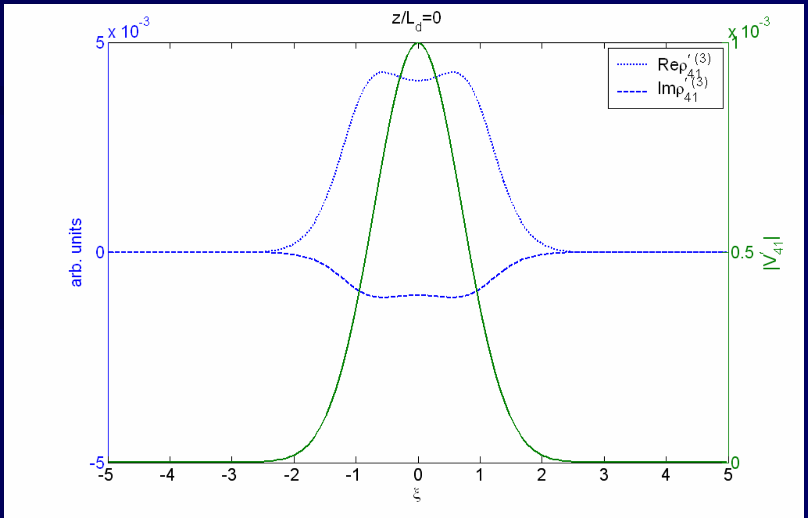
Two strong lasers: strong-weak-strong –weak configuration

$V_{31}=4;\quad V_{42}=4;\quad V_{32}=0.1;\quad V_{41}=0.001;$

$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}=-4;$

$L_L/L_D=1.66\times10^{-3};$



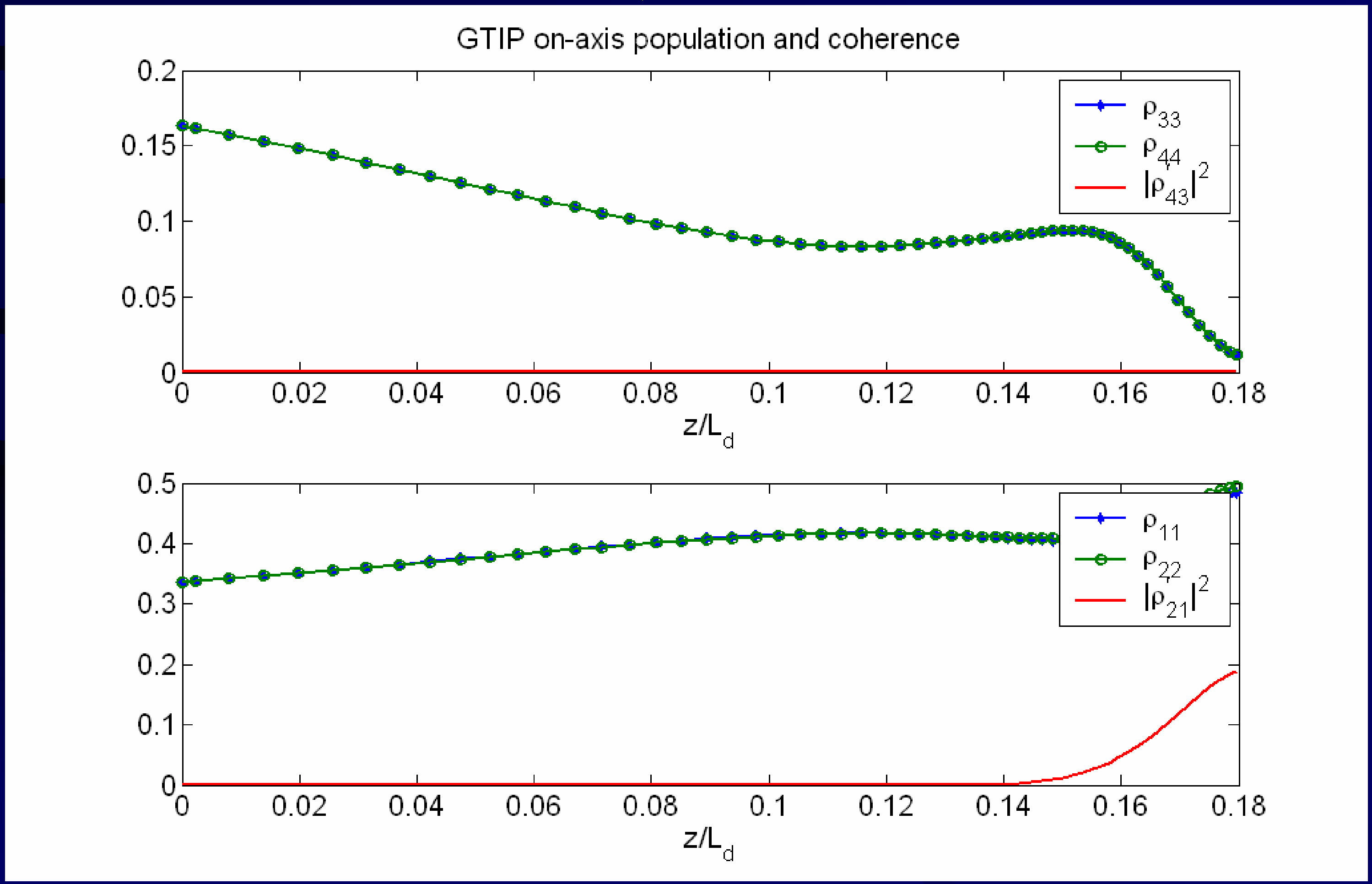


Two strong lasers: strong-weak-strong –weak configuration

$V_{31}=4; \quad V_{42}=4; \quad V_{32}=0.1; \quad V_{41}=0.001;$

$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}=-4;$

$L_L/L_D=1.66\times10^{-3};$

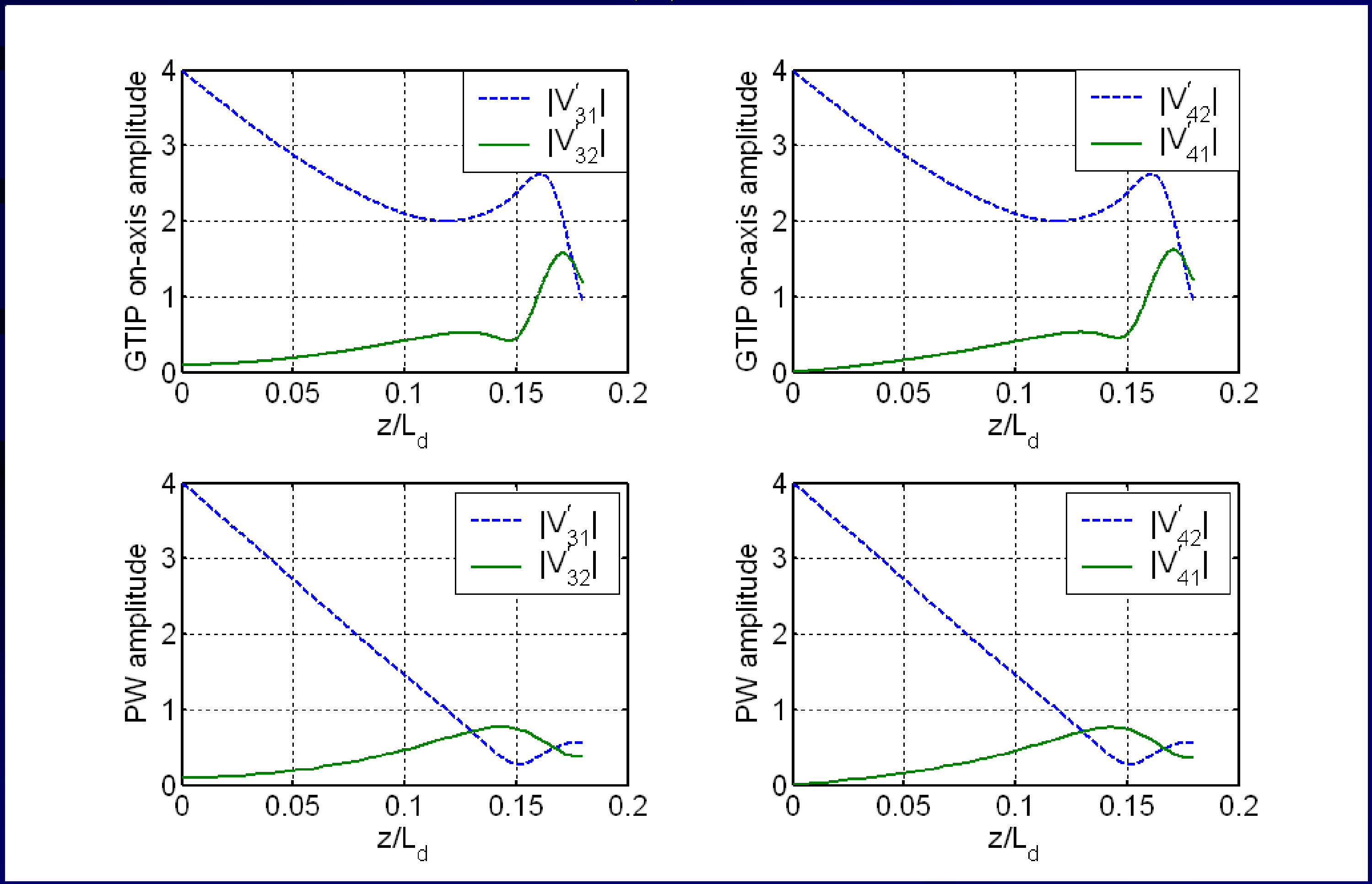


Two strong lasers: strong-weak-strong –weak configuration

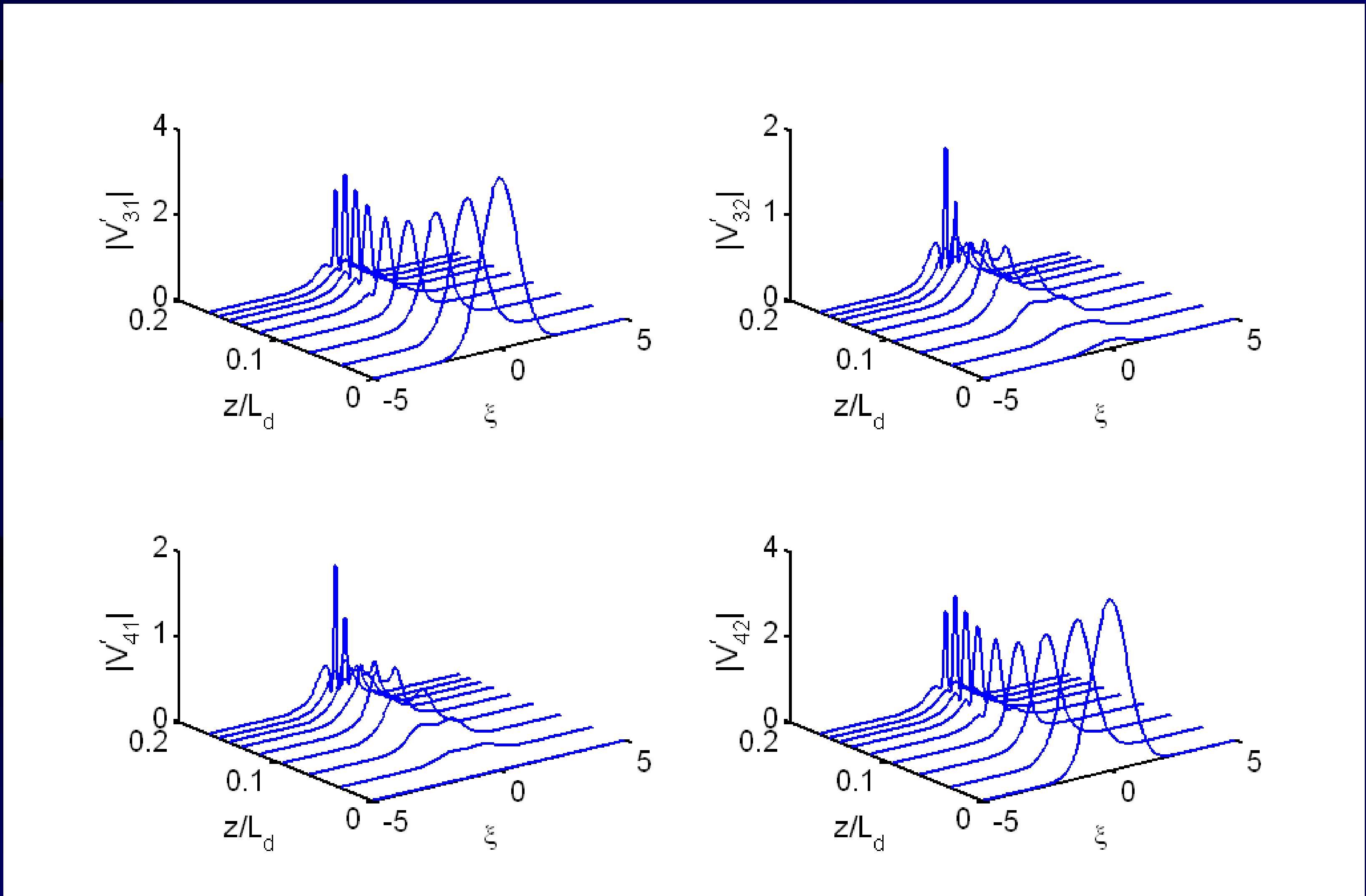
$V_{31}=4;\quad V_{42}=4;\quad V_{32}=0.1;\quad V_{41}=0.001;$

$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}=-4;$

$L_{NL}/L_D=1.66\times10^{-3};$



Focusing on propagation



Phase dependence

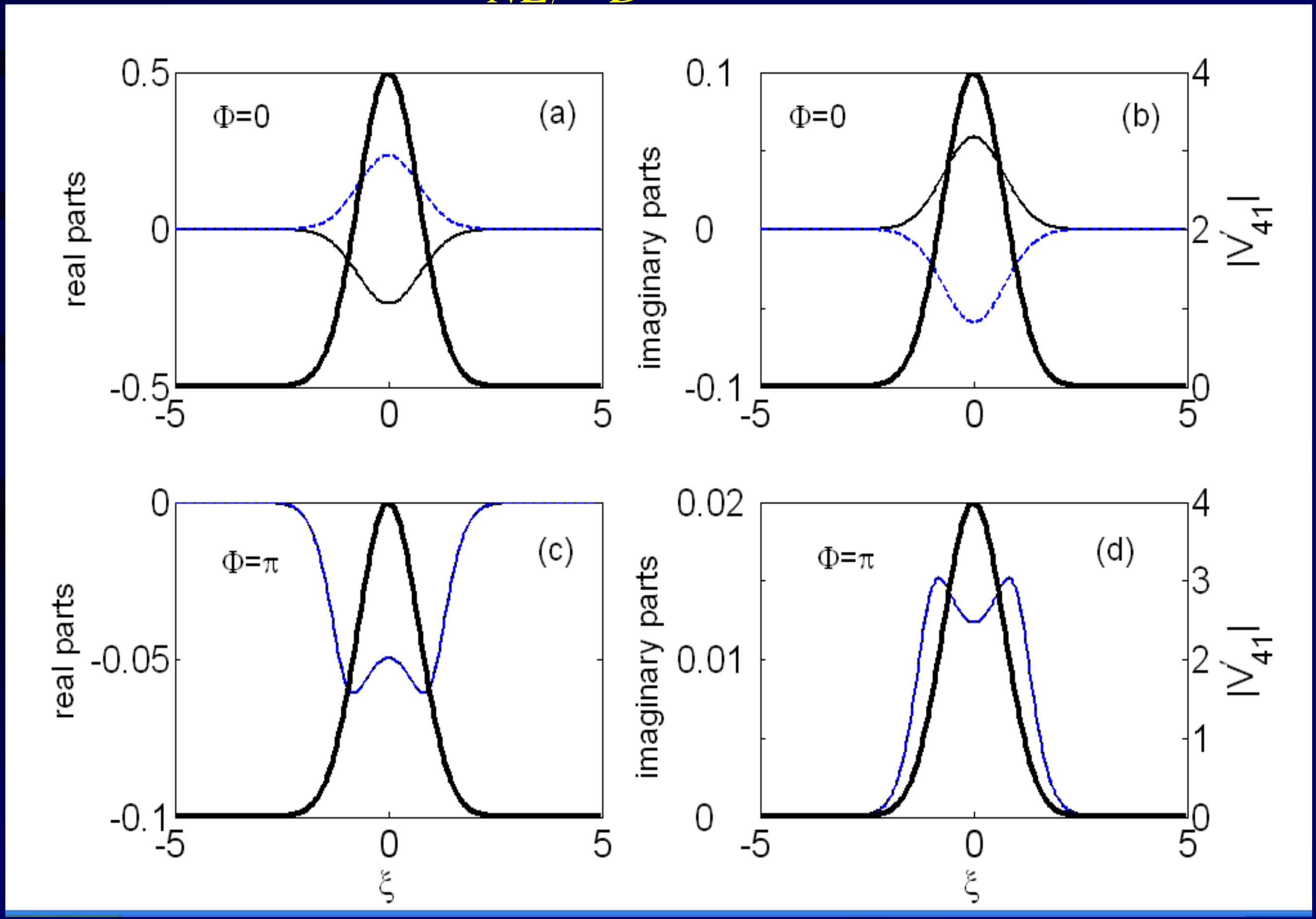
- When field at FWM frequency absent or small at outset, phase is unimportant
- When field at FWM frequency present at outset, dramatic phase effects can be obtained

Four Strong Fields: phase 0 vs. π

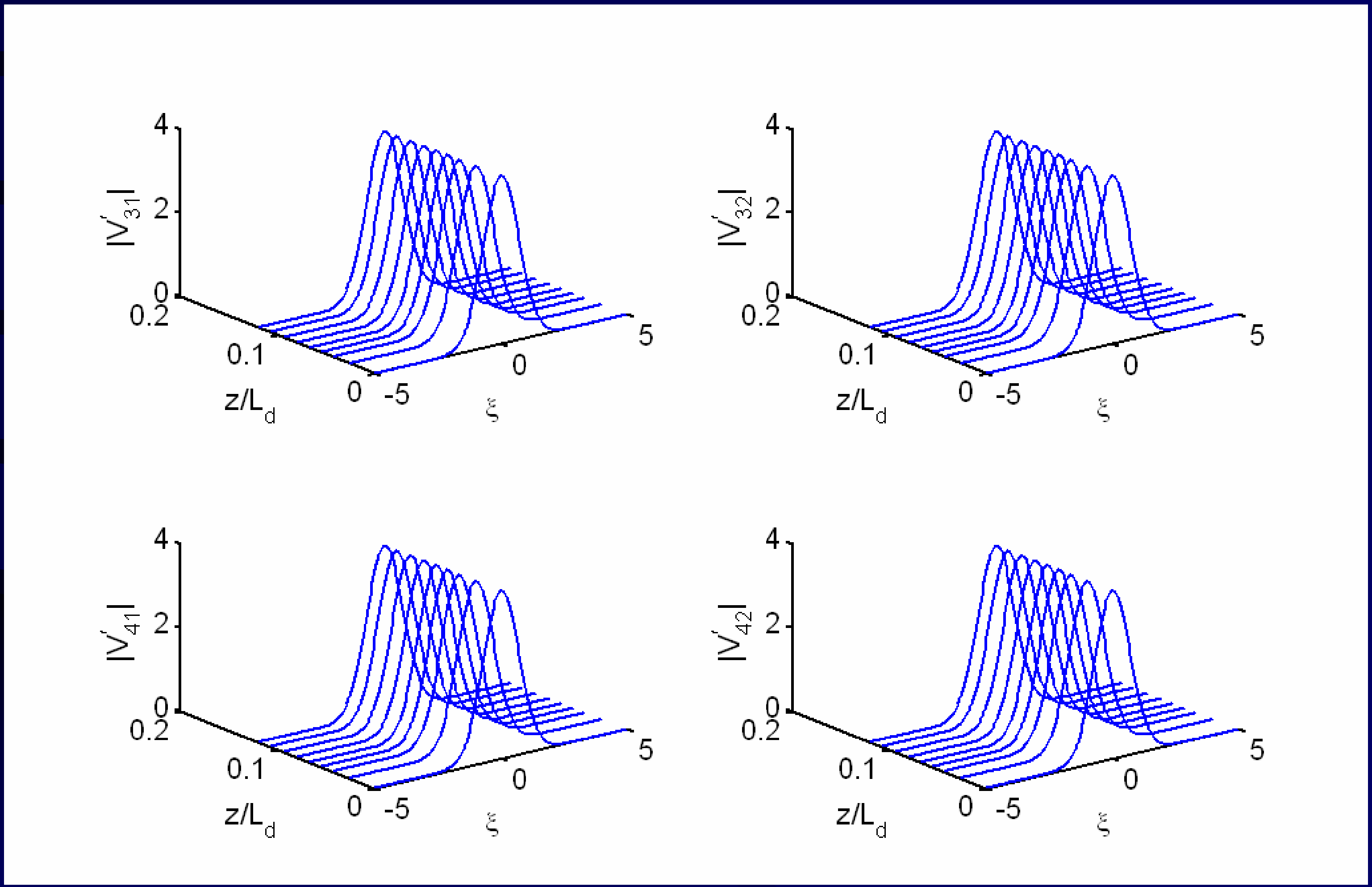
$V_{31}=4; \quad V_{42}=4; \quad V_{32}=4; \quad V_{41}=4;$

$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}=-4;$

$L_{NL}/L_D=1.11\times10^{-3};$



Zero Phase
No Change on Propagation: CPT

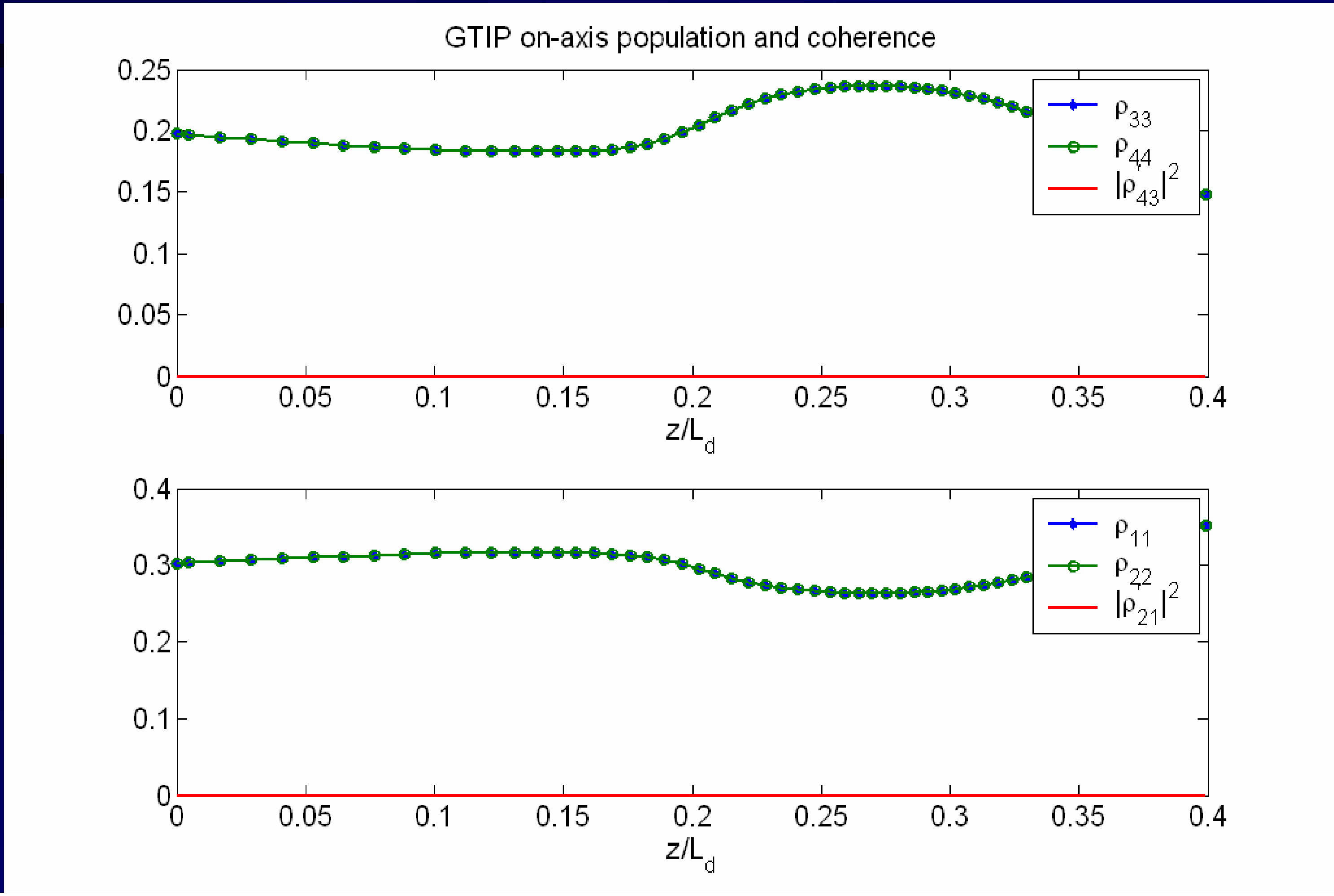


Four strong fields: phase π

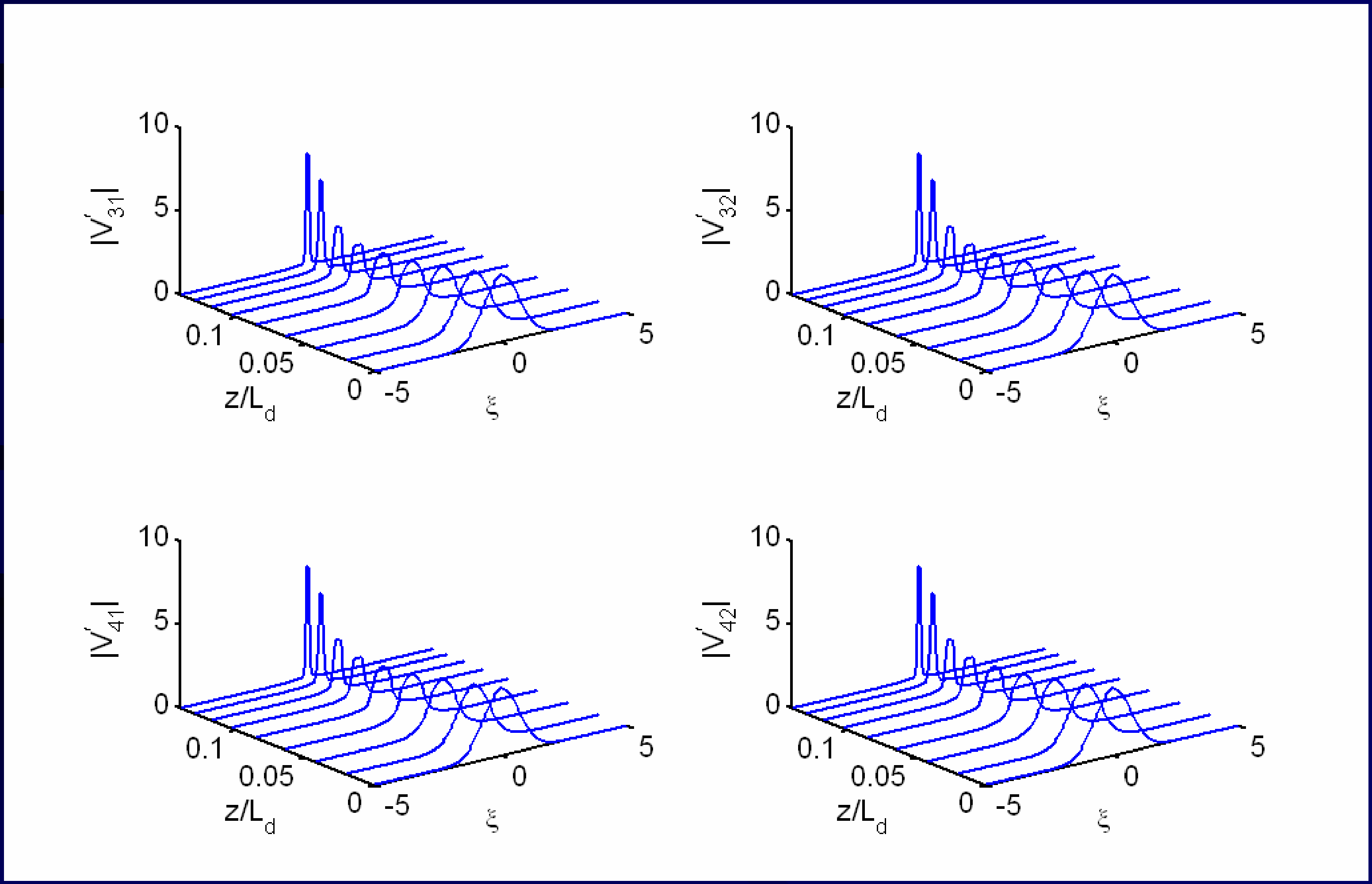
$V_{31}=4; \quad V_{42}=4; \quad V_{32}=4; \quad V_{41}=4;$

$\Delta_{31}=\Delta_{32}=\Delta_{41}=\Delta_{42}=-4;$

$L_{NL}/L_D=1.11\times10^{-3}; \quad \Phi=\pi;$



Phase π
Focusing on propagation: no CPT



Conclusions

- Efficient FWM in double lambda systems can be obtained even before CPT occurs
- It is obtained at short propagation distances
- Focusing can be obtained by blue one-photon detuning
- Often accompanied by ring formation

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