Credit Derivatives Markets

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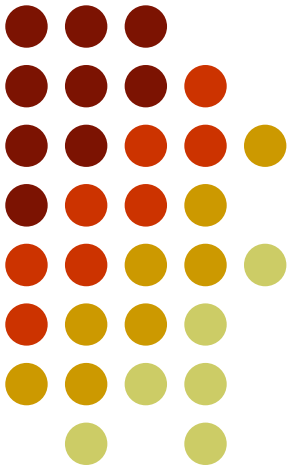
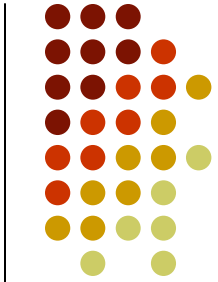
PRMIA/Sungard/Fields/Rotman Meeting

February 7, 2005

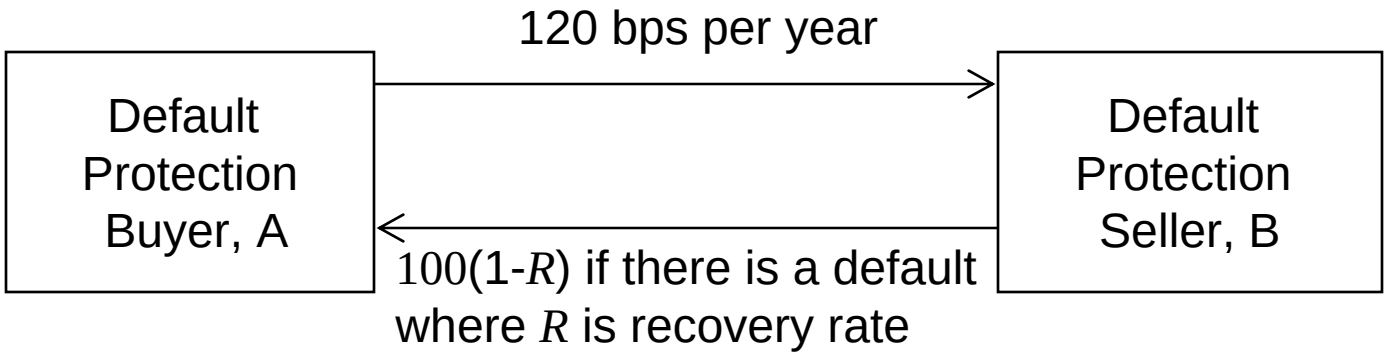


Products that have become important during last 7 years

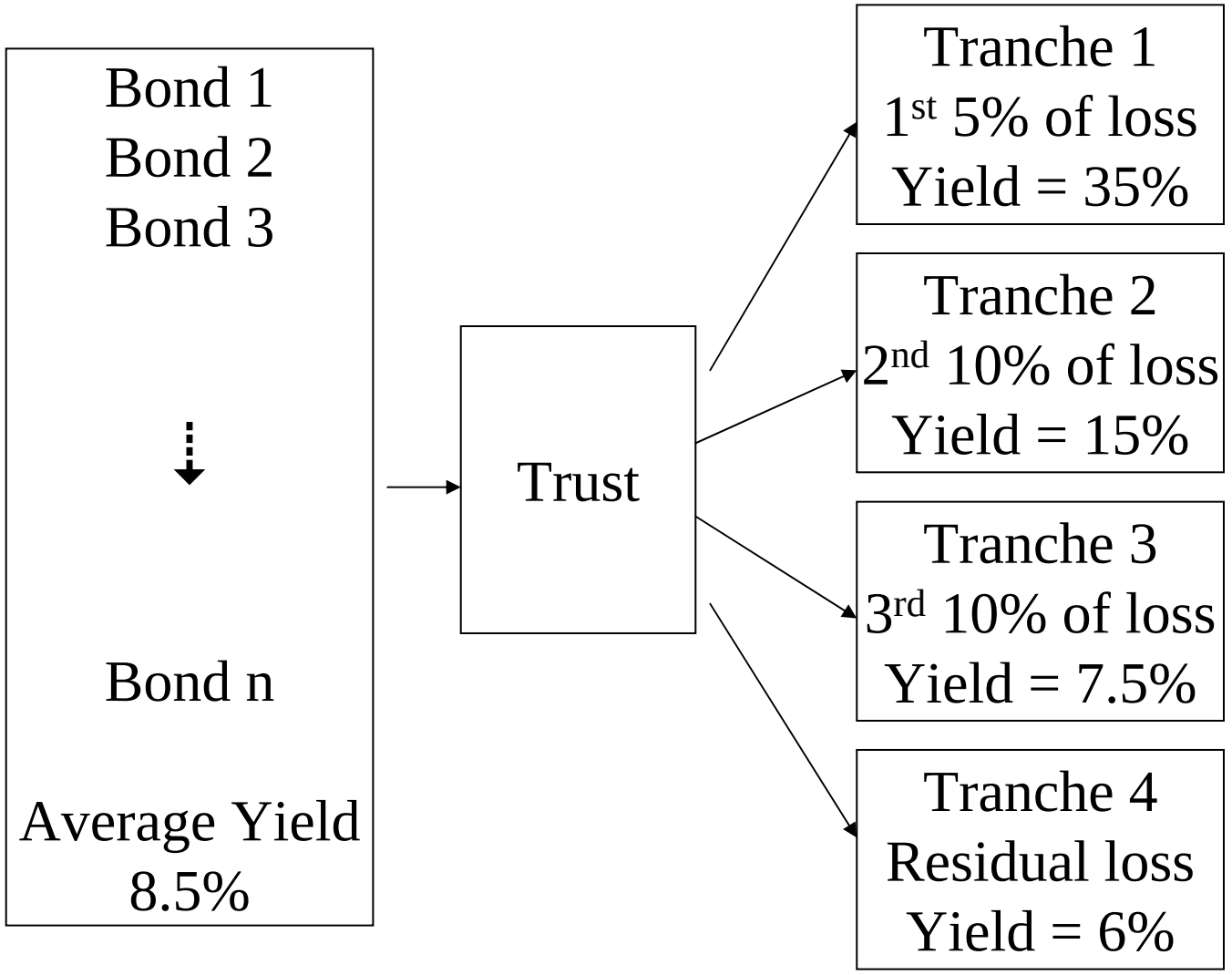


- Credit Default Swaps
- Cash CDOs
- Synthetic CDOs
- Single Tranche Trading
- CDO Squareds

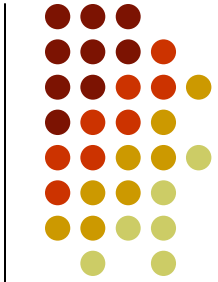
Credit Default Swap



Cash CDO

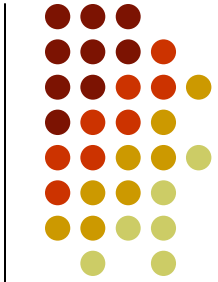


Valuation Issues



- Default probability estimates
- The correlation model

Real world vs risk-neutral default probabilities

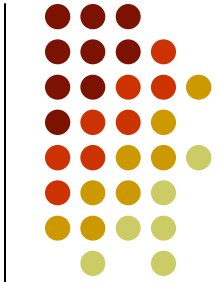


- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities
- The default probabilities backed out of historical data are real-world default probabilities

A comparison

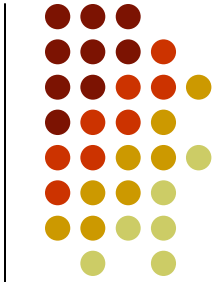
- We calculated 7-year default intensities from Moody's data between 1970 and 2003 (These are real world default probabilities)
- We used Merrill Lynch data (Dec 1996 to July 2004) to estimate average 7-year default intensities from bond prices (these are risk-neutral default intensities)
- We assumed a risk-free rate equal to the 7-year swap rate minus 10 basis points and a recovery rate of 40%

Real World vs Risk Neutral Default Probabilities (7 year averages)



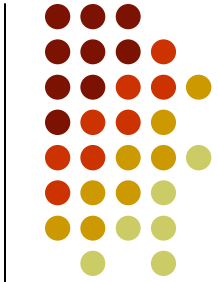
Rating	Real-world default probability per yr (bps)	Risk-neutral default probability per yr (bps)	Ratio	Difference
Aaa	4	67	16.8	63
Aa	6	78	13.0	72
A	13	128	9.8	115
Baa	47	238	5.1	191
Ba	240	507	2.1	267
B	749	902	1.2	153
Caa	1690	2130	1.3	440

Risk Premiums Earned By Bond Traders



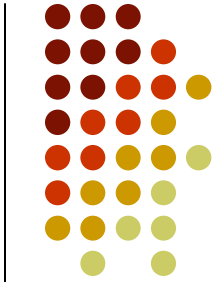
Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	83	43	2	38
Aa	90	43	4	43
A	120	43	8	69
Baa	186	43	28	115
Ba	347	43	144	160
B	585	43	449	93
Caa	1321	43	1014	264

Possible Reasons for These Results



- Corporate bonds are relatively illiquid
- The subjective default probabilities of bond traders may be higher than the estimates from the last Moody's historical data
- Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market

Which World Should We Use?



- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

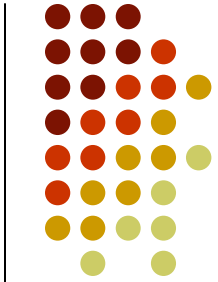
The Correlation Model Used by Market for Index Tranches

- To model default for a basket of companies the market currently uses a Gaussian copula model of time to default where the correlation between each pair of companies is the same
- For company i we define a "default indicator variable" x_i using

$$x_i = aM + \sqrt{1-a^2}\,Z_i$$

  M and the Z_i have independent distributions with zero mean unit variance
- The correlation between x_i and x_j is a^2
- The x_i is mapped to the default time, t_i of the i^{th} company on a "percentile-to-percentile" basis

General Approach to Quasi-Analytic Valuation



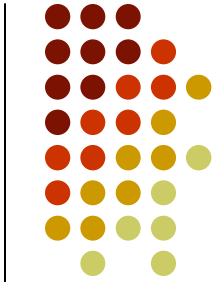
- Calculate cash flows conditional on the value of the factor(s) and then integrate over the probability distribution for the factor(s)
- The integration can be accomplished accurately and efficiently using Gaussian quadrature

Empirical Results for DJ iTraxx EUR Tranches, Aug 4, 2004

(Gaussian copula model)

Tranche	0 - 3%	3 - 6%	6 - 9%	9 - 12%	12 - 22%
Market Quote	27.6%	168	70	43	20
Correlation	Model Quotes				
0.00	44.3%	69	0	0	0
0.05	39.7%	161	10	1	0
0.10	35.4%	222	36	6	0
0.15	31.5%	258	64	18	2
0.20	27.9%	281	90	33	6
0.25	24.5%	294	110	49	11
0.30	21.2%	300	127	64	18
0.40	15.2%	299	151	86	34
Implied Correlation	0.204	0.055	0.161	0.233	0.312
Base Correlation	0.204	0.288	0.337	0.369	0.448

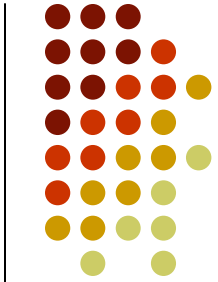
Test Results ($x_i = aM + \sqrt{1-a^2}Z_i$)



Spread tends to reduce for tranches up to a certain level of seniority and increase for tranches beyond that level of seniority when:

- Correlation increases
- Tails of M become heavier
- Tails of Z_i's become less heavy
- There is a negative correlation between recovery rates and the level of defaults
- We move to a two-factor model

Test Results continued



Spread tends to increase for junior and senior tranches and decline for intermediate tranches when:

- Tails of both M and Z_i's are made heavier

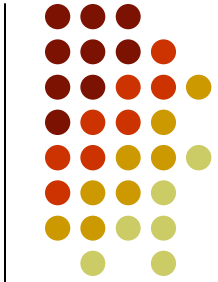
A Variation on the Standard Market Model that fits Market Prices Much Better

- The double t-distribution copula
- In

$$x_i = aM + \sqrt{1-a^2}\,Z_i$$

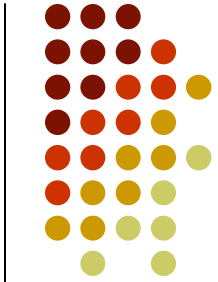
M and the Z_i have t-distributions with a small number of degrees of freedom

Results for iTraxx on Aug 4, 2004 when M and the Z_i Have t-distributions with 4 Degrees of Freedom



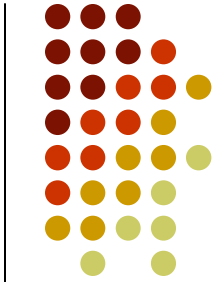
DJ iTraxx EUR					
Tranche	0 - 3%	3 - 6%	6 - 9%	9 - 12%	12 – 22%
Market Quote	27.6%	168	70	43	20
Correlation	Model Quotes				
0.00	43.7%	66	0	0	0
0.05	41.0%	107	9	3	1
0.10	37.9%	133	23	10	4
0.15	34.8%	150	37	18	8
0.20	31.7%	161	49	26	13
0.25	28.6%	167	60	35	18
0.30	25.5%	171	69	42	23
0.40	19.5%	173	84	56	34
Tranche Implied Corr.	0.266	0.258	0.303	0.304	0.270
Base Implied Corr.	0.266	0.266	0.260	0.253	0.241

Simple Test



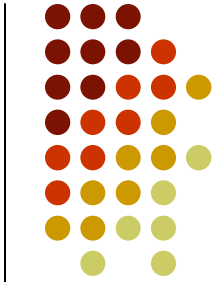
- Choose default probability to match pricing of index
- Choose correlation to match pricing of equity tranche
- Test how well we match other tranches

iTraxx Pricing Errors (bp)
June 21 to August 4, 2004



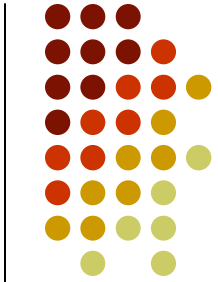
Copula		Tranche				
		0-3%	3-6%	6-9%	9-12%	12-22%
Norm.	Avg.	0.0	123.9	25.5	-8.7	-13.6
	Std. Dev.	0.0	8.6	5.6	4.2	1.1
t_4 - t_4	Avg.	0.0	8.4	-4.7	-5.0	1.8
	Std. Dev.	0.0	6.5	4.4	3.6	1.2

iTraxx Percentage Pricing Errors
June 21 to August 4, 2004



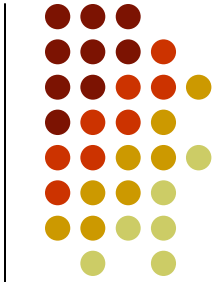
Copula		Tranche				
		0-3%	3-6%	6-9%	9-12%	12-22%
Norm.	Avg.	0.0%	66.2%	32.3%	-17.2%	-63.0%
	Std. Dev.	0.0%	6.2%	8.0%	7.5%	3.8%
t_4 - t_4	Avg.	0.0%	4.6%	-5.6%	-9.7%	8.2%
	Std. Dev.	0.0%	3.4%	5.1%	6.6%	5.7%

Summary



- Important to distinguish real-world and risk-neutral default probabilities
- Implied correlations (particularly base correlations) should be interpreted with care
- A one factor copula model where both M and the Z_i have heavy tails fits market prices well
- For more details see "Bond Prices, Default Probabilities, and Risk premiums" and "The Valuation of a CDO and nth to Default CDS Without Monte Carlo Simulation" on www.rotman.utoronto.ca/~hull

Currrent Research



- Valuation of CDO^2
- A structural model for valuing CDOs