THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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Preserving Stability of Steady States of Dynamical Systems Upon Discretization and a Cure for Instability

The steady states $u(\lambda)$ of parameter dependent dynamical systems

$$\frac{du}{dt} = G(u, \lambda)$$

are (asymptotically) stable if the spectrum of $G_u(u(\lambda), \lambda)$ lies in the left half of the complex plane. Attempts to compute $u(\lambda)$ invariably lead to itterative or discrete time stepping schemes of the form:

$$u_{j+1}(\lambda) = F(u_j(\lambda), \lambda)$$

The procedure converges, $\{u_j(\lambda)\}_0^\infty \to u_*(\lambda)$, provided that the spectrum of $F_u(u_*(\lambda), \lambda)$ is contained in the unit disk about the origin. The fixed point $u_*(\lambda)$ is then said to be stable. We examine all of the standard numerical methods for integrating the dynamical system and obtain conditions under which the stability of $u(\lambda)$ implies the stability of $u_*(\lambda)$. To simplify the analysis we assume that $u_*(\lambda) = u(\lambda)$, an error that is $O(\Delta t^d)$ for some $d^* \geq 1$.

As λ changes the stability of $u(\lambda)$ may be lost and then $u_*(\lambda)$ cannot be stable. So the fixed point iteration above can no longer converge. We briefly show how RPM, the Recursive Projection Method, can restore stability by a minimal change of the iterative scheme.