THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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On a zero-preserving iso-spectral flow

Abstract:

Let Δ be a set of pairs (i, j) of integers between 1 and n which satisfy the following conditions: (1) for $i = 1, 2, ..., n$, the 'diagonal' pair (i, i) is in Δ, and (2) if the pair (i, j) is in Δ then so is the 'symmetric' pair (j, i). We regard Δ as a 'pattern' of non-zero entries for matrices. In particular, let $Sym(n)$ denote the vector space of symmetric, n-by-n, real matrices and let $Sym(\Delta)$ denote the subspace of $Sym(n)$ consisting of the symmetric matrices which are zero outside the pattern Δ; in symbols,

$$Sym(\Delta) := \{X \in Sym(n) | X(i, j) \neq 0 \text{ implies } (i, j) \in \Delta\}.$$

We use the Frobenius inner product which is defined by $\langle X, Y \rangle := Trace(XY^T)$. With a diagonal matrix D, we associate a real-valued 'objective' function $f : Sym(n) \to R$ defined by $f(X) := \langle X - D, X - D \rangle$.

With a symmetric matrix A, we associate the 'iso-spectral' set $Iso(A)$ of all symmetric matrices which have the same eigenvalues as A. By the spectral theorem, we have $Iso(A) = \{QAQ^T | Q \in O(n)\}$ where $O(n)$ denotes the group of orthogonal matrices.

We shall consider the following constrained optimization problem:

Problem: Given $A \in Sym(\Delta)$, minimize $f(X)$ subject to the constraints $X \in Iso(A)$ and $X \in Sym(\Delta)$.

In particular, we shall describe a flow on the surface $Iso(A) \cap Sym(\Delta)$ which has the critical points of f as equilibrium points. Note that the QR-algorithm is closely related to flows - the QR flow, the Toda flow, etc. - like the one we consider here. We have added the zero pattern as an extra constraint.