Catenarity in a class of iterated skew polynomial rings

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Abstract

A ring R is catenary, if, for any two prime ideals $P < Q$ of R,all saturated chains of prime ideals between P and Q have the same length.

Tauvel established the following useful height formula for enveloping algebras of solvable Lie algebras: height(P)+GKdim(R/P)=GKdim(R) for all prime ideals P in R.

Sei-Qwon Oh showed that a class of iterated skew polynomial rings is catenary in the case when certain parameters are not roots of unity.

In this thesis, we state definitions and properties of Gelfand- Kirillov dimension. Then, we explain the abstract properties sufficient to prove a catenarity results i.e. Gabber's method abstracted. Finally, we explain and clarify catenarity and Tauvel's height formula for a class of iterated skew polynomial rings includes the multiparameter quantized Weyl algebra and the coordinate ring of quantum symplectic space and the coordinate ring of quantum Euclidean space. In addition, we introduce some of concepts which is related to Gelfand-Kirillov dimension.

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