Distinguishing groupwise density numbers

Jörg Brendle

November 4, 2007

The notion of groupwise density and the related groupwise density number were introduced by Andreas Blass and have since found many applications.

Recall that a family $\mathcal{D}$ of infinite subsets of ω is called *groupwise dense* if $\mathcal{D}$ is open and for every partition $(I_n : n \in \omega)$ of ω into finite intervals, there is an infinite $B \subseteq \omega$ such that $\bigcup_{n \in B} I_n \in \mathcal{D}$. Further, $\mathcal{D}$ is a *groupwise dense ideal* if it is groupwise dense and closed under finite unions. The *groupwise density number* $\mathfrak{g}$ is the minimal cardinality of a family of groupwise dense families whose intersection is empty. Similarly, $\mathfrak{g}_f$ is the least size of a family of groupwise dense ideals whose intersection is empty. Clearly $\mathfrak{h} \leq \mathfrak{g} \leq \mathfrak{g}_f$ and $\mathfrak{g}_f \leq \mathfrak{d}$ is also easy to see. Furthermore, the consistency of each of $\mathfrak{b} < \mathfrak{g}$ (and, thus, $\mathfrak{h} < \mathfrak{g}$) and $\mathfrak{g}_f < \mathfrak{d}$ is well-known. Answering a question of Heike Mildenberger, we use a finite support iteration of Laver forcing with carefully chosen filters to show the consistency of $\mathfrak{g} < \mathfrak{g}_f$.