



ANTON ALEKSEEV
University of Geneva

Pure spinors on Lie groups

For any manifold M , the direct sum $TM \oplus T^*M$ carries a natural inner product given by the pairing of vectors and covectors. Differential forms on M may be viewed as spinors for the corresponding Clifford bundle, and in particular there is a notion of pure spinor. In this talk, I'll discuss pure spinors and Dirac structures in the case when $M = G$ is a Lie group with a bi-invariant pseudo-Riemannian metric, e.g. G semi-simple. As an application, I'll revisit the theory of quasi-Hamiltonian G -spaces using the pure spinor formalism.

This talk is based on a joint work with H. Bursztyn and E. Meinrenken, arXiv:0709.1452

DENIS AUROUX
Massachusetts Institute of Technology

Mirror symmetry in the complement of an anticanonical divisor

We study the geometry of the moduli space of special Lagrangian submanifolds in the complement of an anticanonical divisor in a compact Kahler manifold, and its relation to mirror symmetry. In particular we show how the Landau-Ginzburg superpotential arises from a count of holomorphic discs, and wall-crossing phenomena lead to so-called quantum corrections. The general features will be illustrated by considering a specific example: the complex projective plane.

HANS DUISTERMAAT
University of Utrecht

QRT maps and elliptic surfaces

A QRT map is a birational transformation of the plane, complex two-dimensional, which is defined, in a very simple manner, in terms of a pencil of biquadratic curves. The generic curves in the pencil, are elliptic complex algebraic curves, and the QRT map is translation on each of these. The curves do not fiber the plane, because there are, counted with multiplicity, 8 base points and each of the curves of the pencil passes through each of the base points. However, if one blows up the base points, one obtains a compact complex surface which is fibered by the curves. and in this way one obtains practically all of the so-called rational elliptic surfaces. On this elliptic surface, the QRT map defines an honest complex analytic diffeomorphism, and its action on the homology can be used to determine, among others, an explicit formula for the number, for each k , of the curves on which the QRT map is periodic of period k . Using analytical methods, one can also



determine the asymptotic structure for large k of the set of the curves on which the QRT map is k -periodic. Viewed as compact four-dimensional manifolds over the real numbers, the elliptic surfaces can also be viewed as symplectic manifolds, fibered by Lagrangian tori, where singular fibers do occur, and actually are basic in the description of the surface and the singularities in various aspects of the QRT map.

MARCO GUALTIERI

Massachusetts Institute of Technology

Generalized complex 4-manifolds

I will describe some recent progress in our understanding of the structure of 4-dimensional generalized complex structures, which may be viewed as symplectic structures which ‘blow up’, in a sense, along loci where they define complex structures. I will explain my construction with Cavalcanti of a generalized complex structure on the triple connect sum of CP^2 with itself, which is neither a complex nor a symplectic manifold.

VICTOR GUILLEMIN

Massachusetts Institute of Technology

Lecture 1: Some theorems in Math I and Physics I revisited

One of the serendipitous pleasures of mathematics is finding new ways of looking at results which everyone is so familiar with that it seems unlikely that anything interesting or revealing can still be extracted from them. In this talk I’ll share with you a few of my own recent encounters of this kind. I’ll mainly concentrate on theorems in Physics I: spectral properties of the 1-D Schroedinger operator. (This will be a lead-in to the topic of my second lecture: Birkhoff canonical forms.)

VICTOR GUILLEMIN

Massachusetts Institute of Technology

Lecture 2: Semi-classical Birkhoff canonical forms

In this lecture I’ll show how the inverse spectral results for the 1-D Schroedinger operator which I talked about yesterday can be sharpened and extended to higher dimensions by generalizing to the setting of quantum mechanics the classical theory of Birkhoff canonical forms.



TAMÁS HAUSEL
Oxford University

Toric Non-Abelian Hodge Theory

First we give an overview of the geometrical and topological aspects of the spaces in the non-Abelian Hodge theory of a curve and their connection with quiver varieties is. Then by concentrating on toric hyperkaehler varieties in place of quiver varieties we construct the toric Betti, De Rham and Dolbeault spaces and prove several of the expected properties of the geometry and topology of these varieties. This is joint work with Nick Proudfoot.

RICHARD MELROSE
Massachusetts Institute of Technology

Semiclassical quantization and index theorems

Semiclassical quantization on the fibres of a fibration gives a direct odd version of the Atiyah-Singer index map. Using the corresponding quantization of projections, which is closely related to the asymptotic morphism approach of Connes and Higson, semiclassical quantization can in turn be reduced, directly, to the usual even (families) index map and also to the algebraic index construction of Fedosov. The semiclassical, odd, version of the Atiyah-Singer index theorem is relatively easy to prove and shows the close relationship between the various known proofs; these ideas come together in the ‘3-twisted’ projective index theorem with Mathai and Singer.

REYER SJAMAAR
Cornell University

Divided difference operators in equivariant cohomology

Divided difference operators were invented by Newton for the purpose of numerical interpolation. More recently, they showed up in work of Demazure and Bernstein-Gelfand-Gelfand on the invariant theory of compact Lie groups and the cohomology of flag varieties. In this talk, which describes joint work with Tara Holm, I will show that they act naturally as bivariant operations on equivariant cohomology.



SHLOMO STERNBERG
Harvard University

Internal supersymmetry and the standard model of elementary particle physics

In this series of talks I will describe some of the work done jointly with Yuval Ne'eman on the use of supersymmetry in elementary particle physics: the use of Quillen's theory of superconnections in dynamics, and, in particular, to determine the Higgs mass, and the use of special representations of $su(m/1)$ to fix the Weinberg angle and to explain the values of the weak isospin and hypercharge of leptons and quarks, and the issue of generations.

SUSAN TOLMAN
University of Illinois at Urbana-Champaign

Simplified formulae for generalized Schubert calculus

If a circle acts in a Hamiltonian fashion on a symplectic manifold, it induces a natural basis in the cohomology of the manifold. A great deal of interesting mathematics has arisen out of studying the combinatorics of one example of this: the Schubert classes in a flag manifold. I will first discuss work with Rebecca Goldin on how some of these ideas extend to give combinatorial formula which work on any symplectic manifold. I will then explain more recent work with Silvia Sabatini on how to simplify these formula so that many fewer paths contribute; in some particularly nice cases, we get positive integral formula.

ALEJANDRO URIBE
University of Michigan

An overview of some recent semiclassical results

I will talk about two unrelated semiclassical results: (1) Asymptotics of eigenvalue clusters for perturbations of the hydrogen atom, and (2) Asymptotics of zeroes of discrete orthogonal polynomials. The first result (joint with C. Villegas-Blas) is a semi-classical analogue of a theorem of Colin de Verdière, Spencer, and Weinstein on eigenvalue clusters for the laplacian plus a potential on the n -sphere, that uses Moser's regularization of the Kepler flow. The second result exhibits a perhaps surprising relationship between geometric quantization and discrete orthogonal polynomials (work in progress).



STEVE ZELDITCH
Johns Hopkins University

Kahler quantization and the homogeneous Monge Ampere equation

A central topic in Kahler geometry is to relate GIT notions on a polarized Kahler manifold (M, ω, L) to transcendental geometric notions such as Calabi extremal metrics in the class of ω . The transcendental analogue of a one-parameter subgroup is a geodesic in the space of Kahler metrics in the fixed class. It is a solution of the homogeneous complex Monge-Ampere equation on $A \times M$ where A is an annulus. I will review recent results of Phong-Sturm and of myself and Jian Song (and work in progress with Yanir Rubinstein) which construct Monge-Ampere geodesics by taking limits of one parameter subgroup geodesics (Bergman geodesics). Thus, Kahler quantization is used to solve Monge Ampere.