Generating Symmetric Tensegrities

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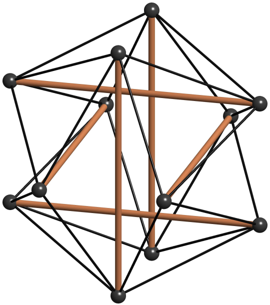

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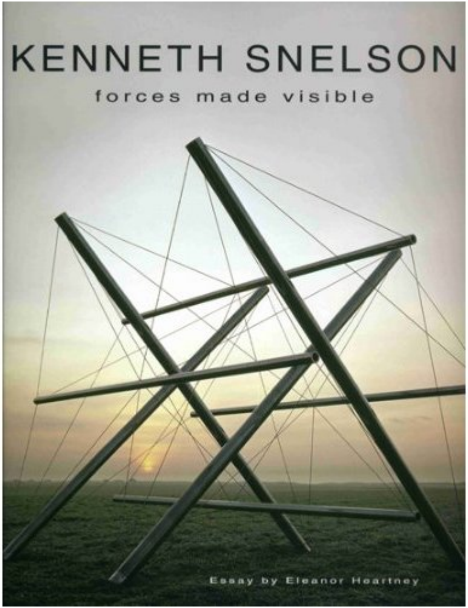
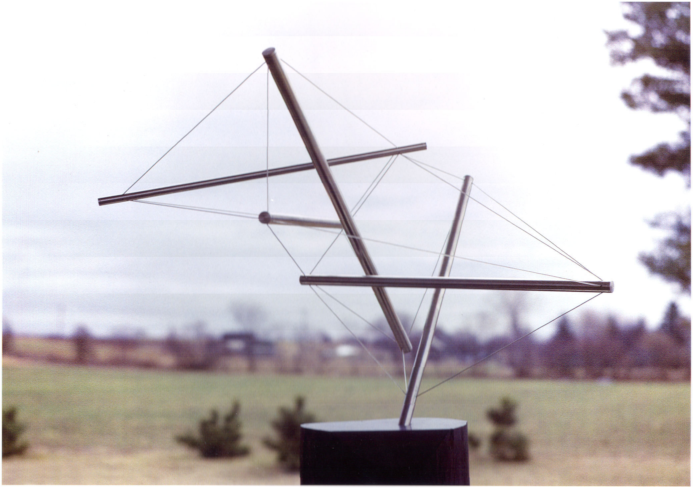


Examples: Kenneth Snelson sculptures



Sag Harbor I, 1965
stainless steel with iron-wood base
13 x 10 x 10 in
33 x 25.5 x 25.5 cm

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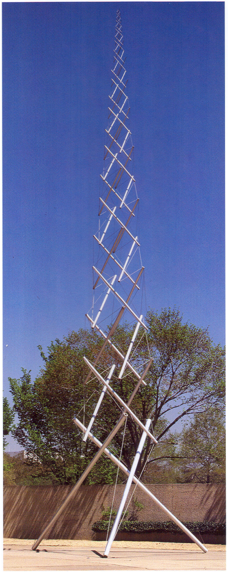

Needle Tower, 1968
aluminum and stainless steel
60 x 20 x 20 ft
18.2 x 6 x 6 m
Collection: Hirshhorn Museum and Sculpture Garden,
Washington, D.C.

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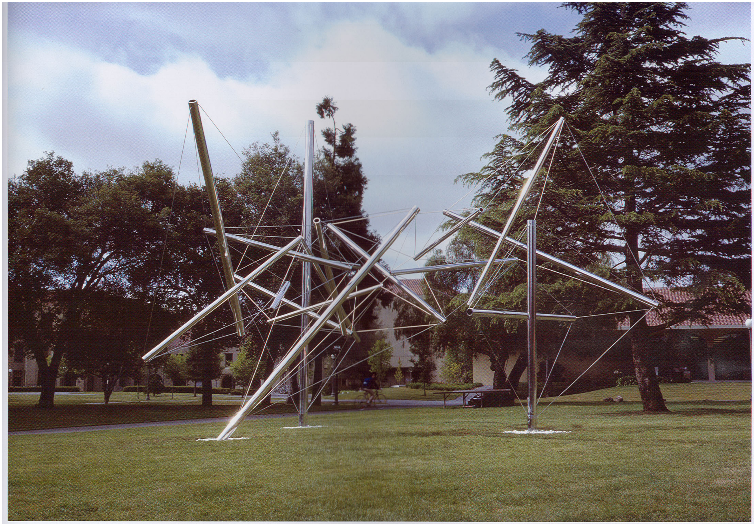

***Mozart I*, 1982**
stainless steel
24 x 24 x 30 ft
7 x 9 x 9 m
Collection: Stanford University, Stanford, CA

Examples: Kenneth Snelson sculptures

Triple Crown, 1991
stainless steel
43 x 85 x 78 ft
13 x 26 x 23 m
Collection: Hallmark, Inc., Kansas City, MO

Examples: Kenneth Snelson sculptures

Dragon, 1999-2000
stainless steel
30.5 x 31 x 12 ft
9.29 x 9.44 x 3.65 m

Some example definitions

R. Buckminster Fuller, in *Synergetics: Explorations in the Geometry of Thinking*

> *Tensegrity describes a structural-relationship principle in which structural shape is guaranteed by the finitely closed, comprehensively continuous, tensional behaviors of the system and not by the discontinuous and exclusively local compressional member behaviors*

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R. Motro, in *Tensegrity: Structural Systems for the Future*

> *A tensegrity system is a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components*

A definition of tensegrity

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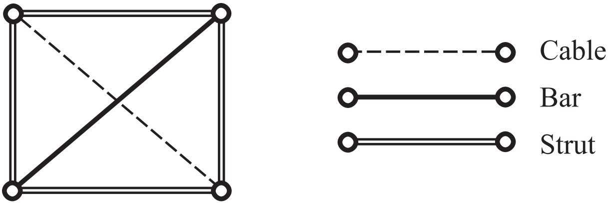
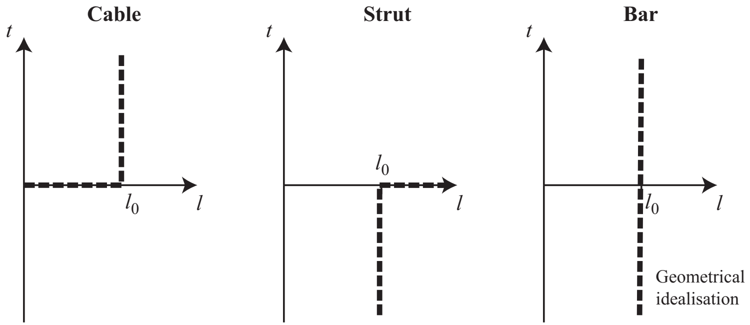


Member idealisation



In a purely geometric contruction, the members are infinitely stiff.

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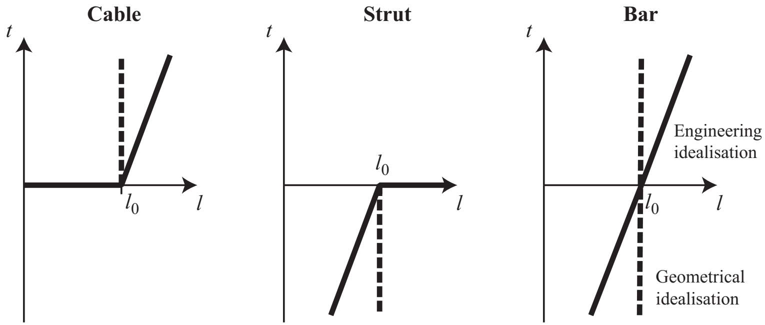

In a purely geometric contruction, the members are infinitely stiff. An engineer might assume that the members have some stiffness k, and:

cable $t = k(l - l_0)$ for $l \geq l_0$, $t = 0$ for $l < l_0$;

strut $t = -k(l_0 - l)$ for $l \leq l_0$, $t = 0$ for $l > l_0$;

bar $t = k(l - l_0)$ for all l.

Why is tensegrity interesting (to a structural engineer)?

Tensegrities are mysterious to a structural engineer, because they may rely on stiffness terms that are neglected (from the very start) in our education of undergraduate engineers.

A starting point for the usual approach to stiffness — Maxwell's constraint counting (1864)

A frame is a system of lines connecting a number of points, and a stiff frame is one in which the distance between any two points cannot be altered without altering the length of one or more of the connecting lines of the frame.

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> *A frame having j joints (points) in space requires in general $3j - 6$ bars (lines) to render it stiff.*

But many tensegrities do not satisfy this condition.

Maxwell's comment on tensegrity

> *In those cases where stiffness can be produced with a smaller number of bars (lines), certain conditions must be fulfilled, rending the case of a maximum or minimum value of one or more of its bars (lines). The stiffness of the frame is of an inferior order, as a small disturbing force may produce a displacement infinite in comparison with itself.*

In fact, the conditions under which Maxwell's exceptional cases occur also permit at least one *state of self-stress* in the frame — internal forces, with no external applied load.

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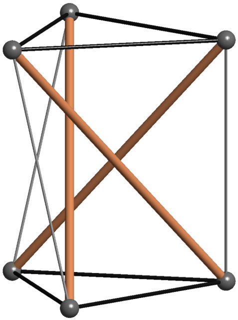

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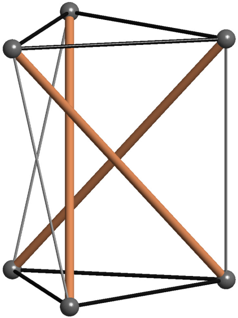

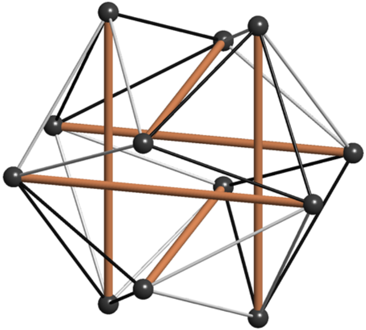

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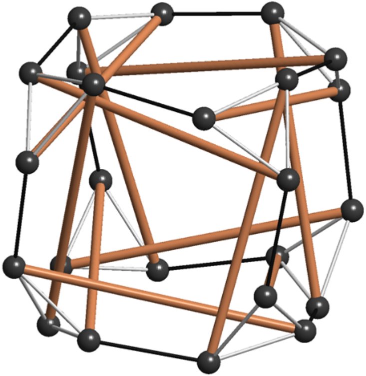

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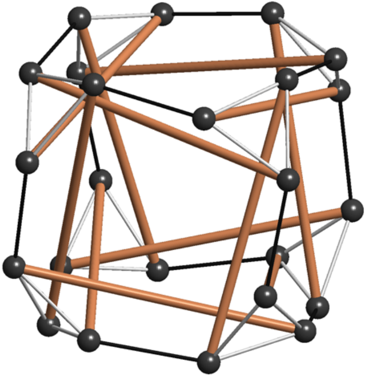
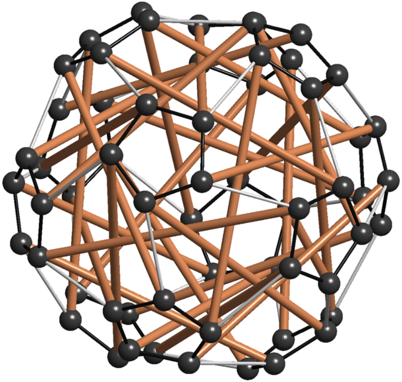

Examples: a new sculpture, the ‘Wigsthorpe’ tensegrity

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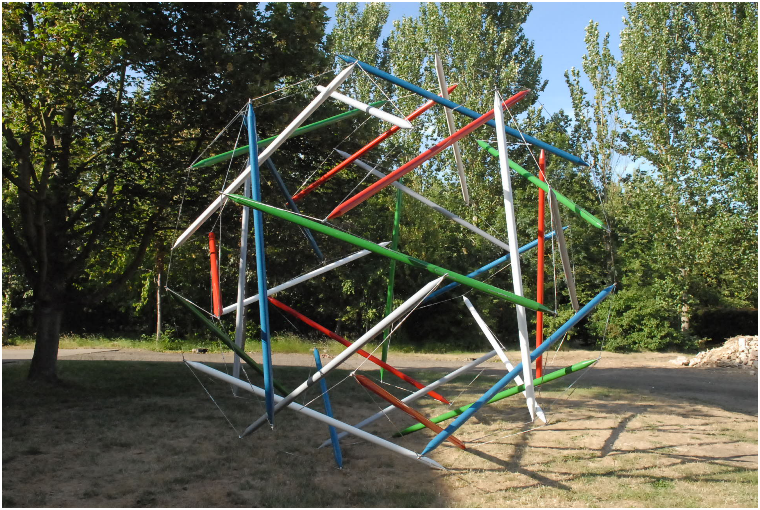

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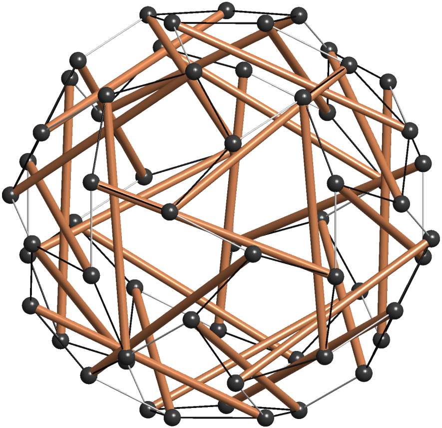

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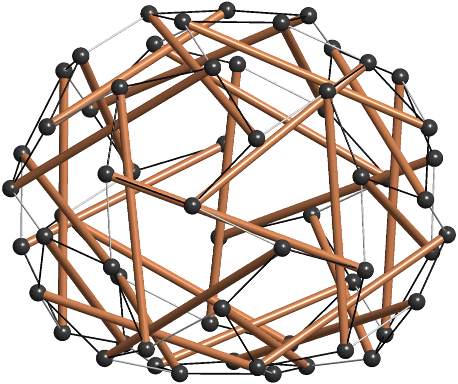

Another source for symmetric tensegrities

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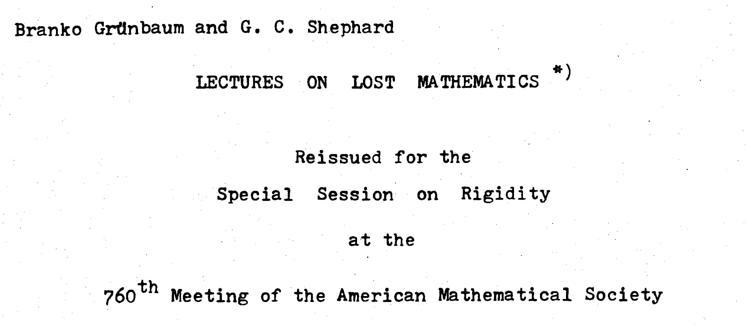


Branko Grünbaum and G. C. Shephard

LECTURES ON LOST MATHEMATICS *)

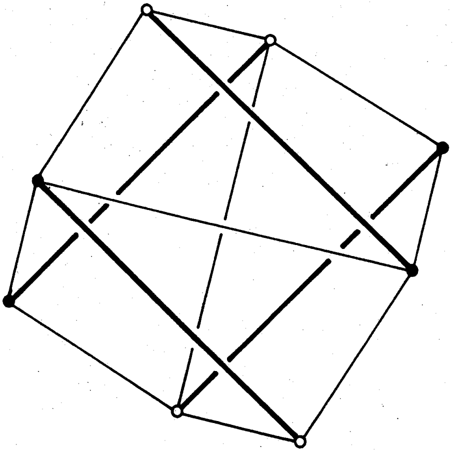
Reissued for the

Special Session on Rigidity

at the

760th Meeting of the American Mathematical Society

Grünbaum examples



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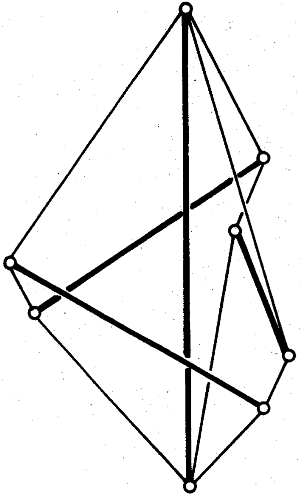

Equilibrium of a tensegrity using the Stress Matrix

If we consider the *force density* ω_{ij} for each member connecting node i to node j to be fixed, we can write

$$\mathbf{S}\mathbf{p} = \mathbf{f}$$

where

$$\mathbf{p} = \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_n \end{bmatrix}$$

is a configuration of the tensegrity, and

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_n \end{bmatrix}$$

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is a self-balancing set of forces applied at the nodes and **S** is the *large stress matrix*.

The *Stress Matrix*

The large stress matrix $\mathbf{S}$ has a very simple formulation in terms of the *small stress matrix* $\boldsymbol{\Omega}$,

$$\mathbf{S} = \boldsymbol{\Omega} \otimes \mathbf{I}$$

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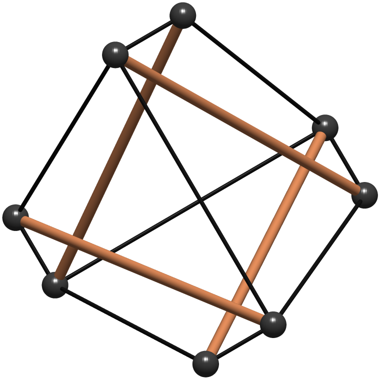
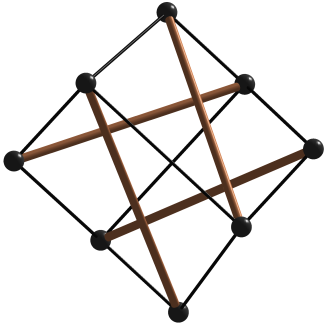
(The small stress matrix is often called the *force density matrix* in engineering literature)

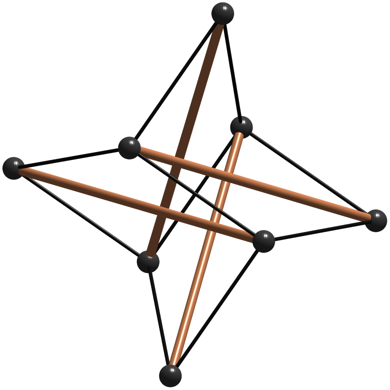
Grünbaum example with $\omega_s = -1$, $\omega_v = 1$ $\omega_h = 4$, $\omega_d = 0.5$



Grünbaum example with $\omega_s = -1$, $\omega_v = 1$ $\omega_h = 2$, $\omega_d = 1$



Grünbaum example with $\omega_s = -1$, $\omega_v = 1$ $\omega_h = 1$, $\omega_d = 2$



and finally . . .

Plug number 1 — book should be complete next year

Frameworks, Tensegrities and Symmetry: Understanding Stable Structures

R. Connelly S.D. Guest

Plug number 2 — 'Rigidity of periodic and symmetric structures', meeting on 23rd, 24th February 2012

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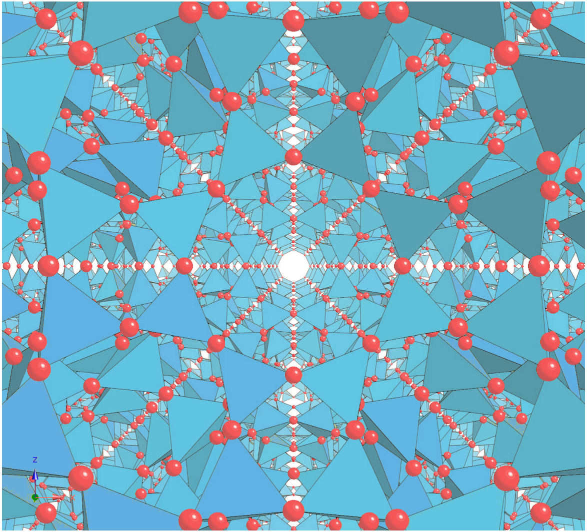

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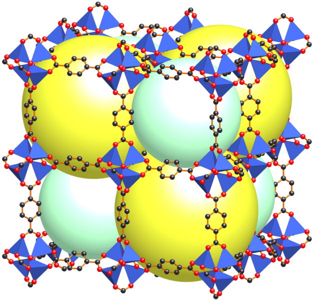

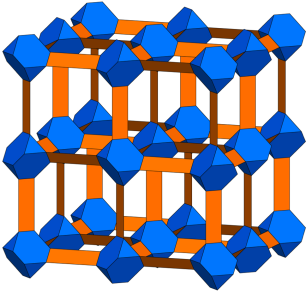

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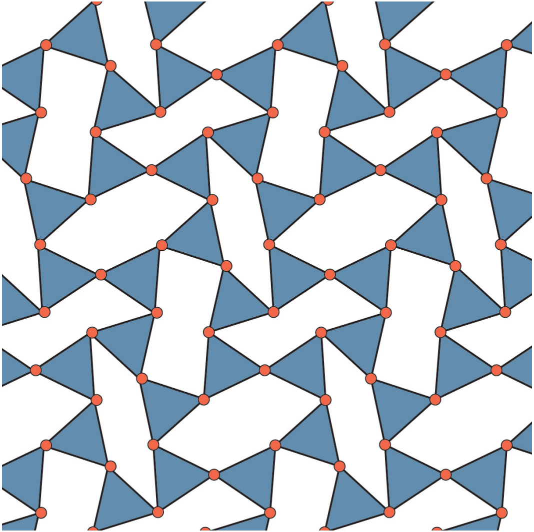

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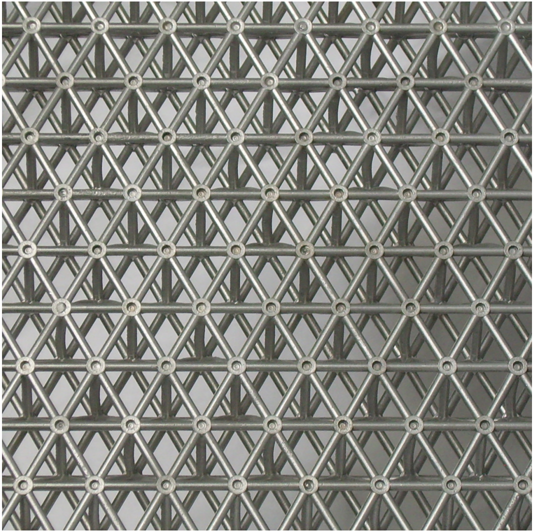

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http://royalsociety.org/events/
Rigidity-of-periodic-and-symmetric-structures/