Feedback of zonal flows on Rossby-wave turbulence driven by small scale instability

Colm Connaughton

Mathematics Institute and Centre for Complexity Science
University of Warwick, UK

Collaborators: B. Nadiga (LANL), S. Nazarenko (Warwick), B. Quinn (Warwick/University College Dublin).

Workshop on Wave Interactions and Turbulence
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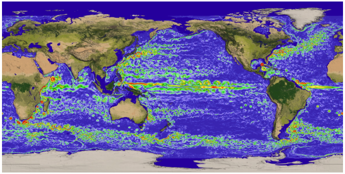
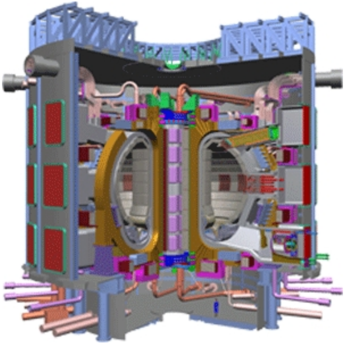
Coexistence of large scale coherent structures and small scale turbulence in geophysical flows

Zonal turbulence on Jupiter (NASA)



Eddy-resolving simulation of Earths oceans (Earth Simulator Center/JAMSTEC)

Zonal Flows and turbulence in magnetised plasmas



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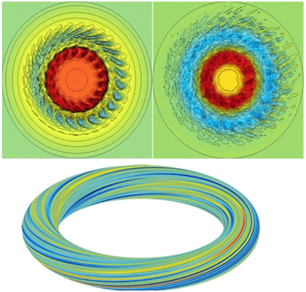


Plasma turbulence (L. Villard)

Outline

1. Introduction: Rossby wave turbulence
2. Generation of large scales by modulational instability
3. Feedback of large scales on small scales

Outline

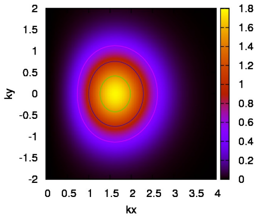
Charney-Hasegawa-Mima equation driven by small scale "instability"

$$(\partial_t - \mathcal{L})\,(\Delta\psi - F\psi) + \beta\,\partial_x\psi + J[\psi, \Delta\psi] = 0.$$

Operator $\mathcal{L}$ has Fourier-space representation

$$\mathcal{L}_{\mathbf{k}} = \gamma_{\mathbf{k}} - \nu k^{2m}$$

which mimics a small scale instability (and hyperviscosity).



- $\gamma_{\mathbf{k}}$ peaked around $(k_f, 0)$.
- Choose $k_f^2 \sim F$.
- Forcing excites meridionally propagating Rossby waves with wavelength comparable to deformation radius.

Large scale – small scale feedback loop

Initialise vorticity field with white noise of very low amplitude. What are the dynamics? The following scenario was proposed by Zakharov and coworkers (1990's):

- Waves having $\gamma_{\mathbf{k}} > 0$ initially grow exponentially.
- When amplitudes get large enough, nonlinearity initiates cascades.
- Inverse cascade transfers energy to large scales leading to zonal jets.
- Jets shear small scale waves generating negative feedback. "Switches off" the forcing.

This talk: is this what really happens?

Weak and strong turbulence regimes

Nonlinearity measured by dimensionless amplitude:

$$M = \frac{\Psi_0 k^3}{\beta}$$

where k typical scale, Ψ_0 is typical amplitude. Two limits:

- $M \gg 1$: Euler limit.
- $M \ll 1$: Wave turbulence limit.

Wave turbulence limit is analytically tractable:

- Evolution of turbulence spectrum described by a closed kinetic equation.
- Kinetic equation has exact stationary solutions describing cascades.

Wave turbulence kinetic equation

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = 4\pi \int \left| V^{\mathbf{k}}_{\mathbf{q}\,\mathbf{r}} \right|^2 \delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})\times$$
$$[n_{\mathbf{q}} n_{\mathbf{r}} - n_{\mathbf{k}} n_{\mathbf{q}}\,\mathrm{sgn}(\omega_{\mathbf{k}}\omega_{\mathbf{r}}) - n_{\mathbf{k}} n_{\mathbf{r}}\,\mathrm{sgn}(\omega_{\mathbf{k}}\omega_{\mathbf{q}})]\, d\mathbf{q}d\mathbf{r} + \gamma_{\mathbf{k}}\, n_{\mathbf{k}}.$$

where

$$\omega_{\mathbf{k}} = -\frac{\beta\, k_x}{k^2+F}$$

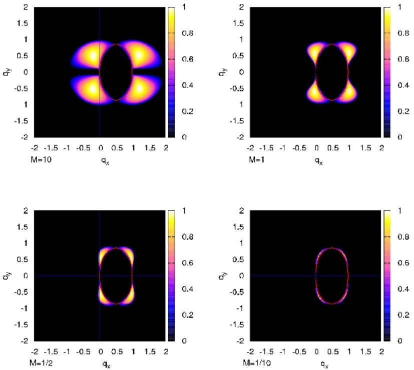
$$V^{\mathbf{k}}_{\mathbf{q}\,\mathbf{r}} = \frac{i}{2}\sqrt{\beta\,|k_x q_x r_x|}\left(\frac{q_y}{q^2+F} + \frac{r_y}{r^2+F} - \frac{k_y}{k^2+F}\right).$$

Problem: stationary Kolmogorov solution describing the inverse cascade is non-local.

Outline

Modulational instability of Rossby waves

Small scale Rossby waves are unstable to large scale modulations (Lorenz 1972, Gill 1973)



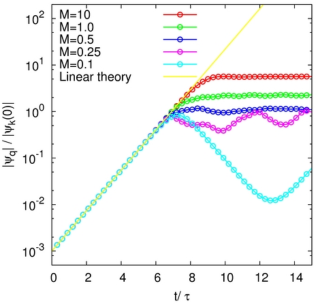
$\mathbf{p} = (1, 0).$

- In wave turbulence limit, $M \ll 1$, unstable perturbations concentrate on resonant manifold:

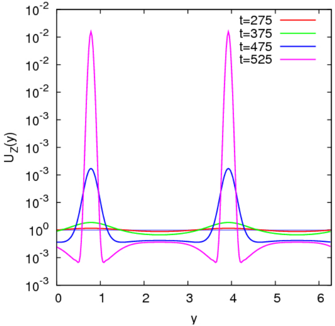
$$\begin{aligned} \mathbf{p} &= \mathbf{q} + \mathbf{r} \\ \omega(\mathbf{p}) &= \omega(\mathbf{q}) + \omega(\mathbf{p}_-). \end{aligned}$$

- Perturbations with fastest growth rate become close to zonal for $M \ll 1$.

Linear stage of modulational instability



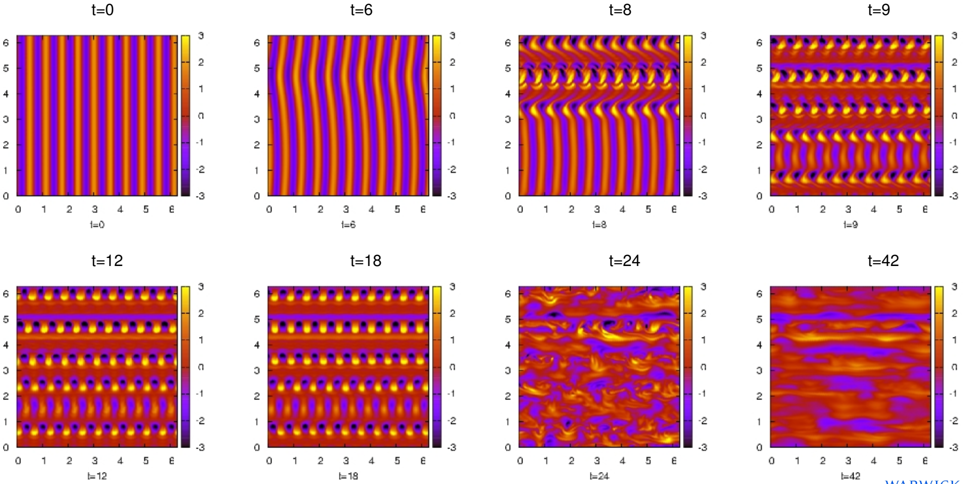
Growth of $|\psi_{\mathbf{q}}|^2$ compared to predictions of linear stability.



Zonal velocity profile:

$$U_Z(y) = \frac{1}{2\pi} \int_0^{2\pi} v_x(x,y)\, dx$$

Generation of jets by modulational instability



Outline

Distortion of small scales by large scales in weakly nonlinear regime

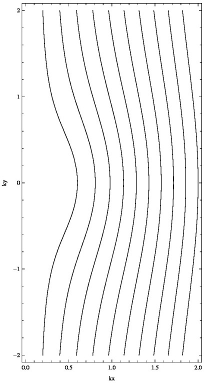
Modulational instability generates large scales directly from small scale waves (non-locality).
Subsequent evolution of the small scales can be described by a nonlocal turbulence theory:

- Assume major main contribution to collision integral for small scales comes from modes **q** having $q \ll k$ (scale separation).
- Taylor expand in **q**. Leading order equation for small scales is an anisotropic diffusion equation in **k**-space:

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = \frac{\partial}{\partial k_i} S_{ij}(k_1, k_2) \frac{\partial n_{\mathbf{k}}}{\partial k_j}.$$

- Diffusion tensor, S_{ij}, depends on structure of the large scales: more intense zonal flows give faster diffusion.

More on the spectral diffusion approximation

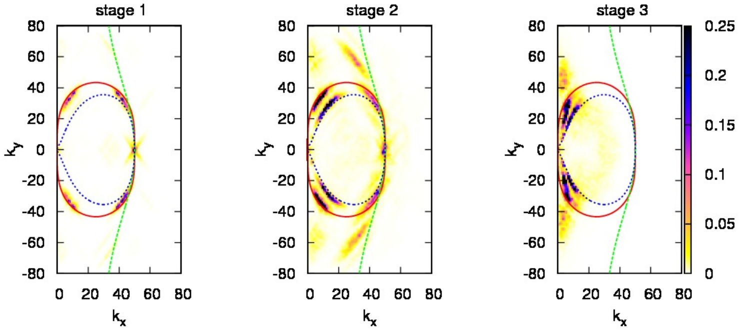


- Diffusion tensor, S_{ij} turns out to be *degenerate*
- A change of variables shows that diffusion is along 1-d curves:

$$\Omega = \frac{\beta\, k^2\, k_x}{k^2 + F} = const$$

- *Wave-action*, $n_{\mathbf{k}}$, of a small scale wavepackets is conserved by this motion.
- Energy, $\omega_{\mathbf{k}}\, n_{\mathbf{k}}$, of small scale wavepackets is not conserved.

Numerical view of spectral transport



Wave-action diffuses along the Ω curves from the forcing region until it is dissipated at large wave-numbers.

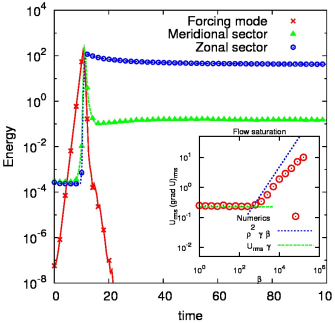
Mechanism for saturation of the large scales

- Energy lost by the small scales is transferred to the large scale zonal flows which grow more intense.
- More intense large scales results in a larger diffusion coefficient.
- Increases the rate of dissipation of the small scale wave-action $\to$ negative feedback.

Prediction:

Growth of the large scales should suppress the small scale instability which turns off the energy source for the large scales leading to a saturation of the large scales *without large scale dissipation*.

Numerical observations of feedback loop



- Feedback loop is very evident in weakly nonlinear regime.
- Scaling arguments allow one to predict the saturation level of the large scales in terms of the small scale growth rate:

$$[U(\nabla U)]_{\mathrm{LS}} \sim \gamma\,\beta \qquad M \ll 1$$

$$[U(\nabla U)]_{\mathrm{LS}} \sim \gamma\,U \qquad M \gg 1$$

These estimates are reasonably supported by numerics - especially the cross-over from the weak to strong turbulence regimes (see inset).

Conclusions

- The CHM equation driven by “instability” forcing behaves differently than one might expect.
- There is no inverse cascade but rather direct generation of large scales by modulational instability (this could change with broadband forcing).
- Evolution of subsequent turbulence is non-local in scale and partially tractable analytically in the wave turbulence limit.
- A negative feedback loop is set up whereby the growth of the small scales is suppressed by the large scales leading to a saturation of the large scales without any large scale dissipation mechanism.

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