Effect of sea-state on upper ocean mixing and dynamics

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INTRODUCTION

In this talk I would like to illustrate that ocean waves play an important role in the interaction between atmosphere and ocean.

Ocean waves play a role in air-sea **momentum transfer** and in **upper ocean mixing**. The associated Stokes drift combined with the earth's rotation results in a additional force on the mean ocean circulation: the **Stokes-Coriolis force**. Also, momentum transfer and the sea state are affected by surface currents, hence it makes sense to introduce a **three-way coupling** between atmosphere, ocean circulation and surface waves. The end result is **one model for the geosphere**. At ECMWF, a first version of such a model will be introduced shortly in the ensemble prediction system and in the monthly forecasting system.

In this context I thought it would be appropriate to revisit the question what are the 'correct' evolution equations for ocean and atmosphere in the presence of ocean waves. This is basically a wave, mean-flow interaction problem and I will concentrate here mainly on the **Stokes-Coriolis force**.

Today, I discuss briefly the following items:

- **Conservation and the surface drift**

  Following Phillips (1977), I introduce the subject by giving for a **rotating earth** the relevant conservation laws for the total velocity which is the sum of the ocean velocity and the surface drift. Next, we obtain the conservation laws based on the ocean circulation velocity and then an additional force arises, the **Stokes-Coriolis force**. In order to justify these laws we need to look at the continuous momentum equations.

  An important role is played by the **surface drift**, where it is noted that the total momentum in the surface drift equals the total momentum in the Stokes drift. For linear waves the surface drift is singular!

- **Quasi-linear equations for the Ocean**

  In order to accommodate in an easy manner effects of the surface drift I will treat **air and water as one fluid**. I will consider arbitrary, but stable density profiles and in the final result I will choose the appropriate density profile that gives surface gravity waves. Thus, I look at internal gravity waves of which surface gravity waves are a special case.

  Quasi-linear approximation: In order to evaluate the effect of waves on the ocean circulation one needs to determine the wave-induced stresses such as $-\langle \delta u \delta v \rangle$ and $-\langle \delta v \delta w \rangle$. These are obtained using the linear wave solutions.

  In particular, on a rotating, deep ocean there is a finite **transverse stress** $-\langle \delta v \delta w \rangle$ which is proportional to the cross product of the coriolis parameter and the difference between Stokes drift and surface drift. **Ergo**, the integral over depth of this stress vanishes.

- **Impact of Stokes-Coriolis forcing**

  I find that in order to understand the momentum balance of total circulation the surface drift plays an essential role. On the other hand, only considering the ocean circulation it is indeed true that there is a Stokes-Coriolis force. In the steady state this force plays an important role in the Ekman turning of the ocean current.

  Additional evidence for the importance of the Stokes-Coriolis force is given by presenting results from 30-year simulations with the Ocean circulation model **NEMO** (Nucleus for European Modelling of the Ocean). Systematic impact on the sea surface temperature field is found.

- **Upper-Ocean Mixing**

  Upper ocean mixing is to a large extent caused by breaking, ocean waves. As a consequence there is an energy flux Φ_{oc} from atmosphere to ocean. It is given by

  $$\Phi_{oc} = m\rho_a u_*^3,$$

  where m depends on the sea state. Wave breaking and its associated mixing penetrates into the ocean at a scale of the significant wave height H_S. In addition, Langmuir turbulence penetrates deeper into the ocean with a scale of the typical wavelength of the surface waves.

  Developed a simple scheme to model these effects and applied it to the diurnal cycle in SST.

  This Scheme is also applied to the NEMO model. In the present version of NEMO there are only averaged sea state effects included, hence m is constant. Here, it is shown that when actual sea state effects are included this will have an impact on the mean SST field.

CONSERVATION OF MASS AND MOMENTUM

Consider an incompressible fluid (water) in a constant gravitational field on a rotating earth. Let the body of water with air above it be of infinite extent in the horizontal while in the vertical it extends from $z = -D$ (with D the water depth) to $z = \eta$, with $\eta(x,y,t)$ the unknown surface elevation. Let us assume that the water motion is governed by the continuity equation

$$\frac{\partial}{\partial t}\rho + \nabla.(\rho\mathbf{u}) = 0. \tag{1}$$

and the momentum equation

$$\frac{\partial}{\partial t}\rho\mathbf{u} + \nabla.\rho\mathbf{u}\mathbf{u} = -\nabla p + \rho\mathbf{g} + \rho\mathbf{u}\times\mathbf{f}. \tag{2}$$

These equations apply to the domain $-D < z < \eta$ and the boundary conditions are

$$z = \eta(x,y,t):\ \frac{\partial}{\partial t}\eta + \mathbf{u}.\nabla_h\eta = w,\ p = p_a, \tag{3}$$

where p_a is the given air pressure at the sea surface and $\nabla_h = (\partial/\partial x, \partial/\partial y)$ is the horizontal gradient operator.

At the flat bottom $D = D_0$ we impose the condition that no fluid penetrates the bottom

$$z = -D: \; w = 0. \tag{4}$$

Conservation laws for the mean surface elevation ζ and the mean horizontal velocity $\mathbf{U}$ may now be obtained by integration of the continuity equation and the momentum equation over the depth of the water, followed by a suitable ensemble averaging. The ensemble average $\langle . \rangle$ is supposed to filter the linear gravity wave motion. Here, the mean surface elevation ζ is defined as

$$\zeta = \langle \eta \rangle, \tag{5}$$

while the mean horizontal velocity $\mathbf{U}$ follows from

$$\mathbf{U} = \frac{\mathbf{P}}{\rho h}, \tag{6}$$

with $h = D + \zeta$ the slowly varying water depth.

Note that $\mathbf{P}$ is the *total* mass flux

$$\mathbf{P} = \langle \int_{-D}^{\eta} dz\, \rho \mathbf{u} \rangle, \tag{7}$$

i.e., it consists of the sum of the water column mean $\mathbf{P}^m$ and the surface layer mean $\mathbf{P}^w$, defined as (Hasselmann, 1971)

$$\mathbf{P}^m = \langle \int_{-D}^{\zeta} dz\, \rho \mathbf{u} \rangle, \;\; \mathbf{P}^w = \langle \int_{\zeta}^{\eta} dz\, \rho \mathbf{u} \rangle. \tag{8}$$

In the linear approximation the surface layer mean mass flux may be expressed in terms of the wave spectrum $F(\mathbf{k})$

$$\mathbf{P}^w = \rho g \int d\mathbf{k}\, \mathbf{l}\, F/c, \tag{9}$$

where c is the phase speed of the gravity waves and $\mathbf{l} = \mathbf{k}/k$ is a unit vector pointing in the direction of the wave propagation. As a consequence, the mean horizontal velocity $\mathbf{U}$ is the sum of the ocean circulation velocity $\mathbf{U}_c$ and the wave-induced drift $\mathbf{U}_{surf}$,

$$\mathbf{U} = \mathbf{U}_c + \mathbf{U}_{surf}. \tag{10}$$

Note that the momentum in the mean surface drift equals the one of the Stokes drift.

The conservation laws become (Mastenbroek et al, 1993)

$$\frac{\partial}{\partial t}\zeta + \nabla_h.\left(h\mathbf{U}\right) = 0, \tag{11}$$

and

$$\left(\frac{\partial}{\partial t} + \mathbf{U}.\nabla_h\right)\mathbf{U} + g\nabla_h\zeta + \frac{1}{\rho}\nabla_h p_a = \mathbf{U}\times\mathbf{f} + \frac{\tau_a - \tau_b}{\rho h} - \frac{1}{\rho h}\nabla_h.\mathsf{S}, \tag{12}$$

where τ_a and τ_b represent the atmospheric surface stress and the bottom stress. The radiation stress tensor S represents the contribution of the wave motions to the mean horizontal flux of horizontal momentum. In terms of the wave spectrum it can be shown to be given by

$$S_{ij} = \rho g \int d\mathbf{k}\left\{\frac{v_g}{c}l_i l_j + \left(\frac{v_g}{c} - \frac{1}{2}\right)\delta_{ij}\right\}F\left(\mathbf{k}\right). \tag{13}$$

Note that the first term corresponds to advection of wave momentum, while the second term consists of a combination of contributions from the wave-induced pressure and the wave-induced stress (Phillips, 1977).

On a rotating earth the above result needs to be justified, therefore the need to consider the full momentum equations.

So far we have not encountered the Stokes-Coriolis force. This force is found if one concentrates on the ocean circulation velocity $\mathbf{U}_c$ as is common practice in oceanography.

To that end one eliminates from (12) the rate of change in time of the wave momentum using the energy balance equation for surface gravity waves. This reads

$$\frac{\partial}{\partial t}F + \frac{\partial}{\partial \mathbf{x}} \cdot \left(\mathbf{v}_g F\right) = S_{in} + S_{nl} + S_{diss} + S_{bot},$$

where $F = F(\mathbf{k})$ is the two-dimensional wave spectrum and $\mathbf{v}_g$ the group velocity. The source functions described the generation of waves by wind (S_{in}), the dissipation of ocean waves by e.g. wave breaking (S_{diss}) and the energy/momentum conserving resonant four-wave interactions (S_{nl}).

Noting that wave momentum is wave energy divided by the phase speed of the waves, one obtains the rate of change of total wave momentum by dividing the energy balance equation by the phase speed $c = \omega/k$ and by integration over wavenumber $\mathbf{k}$. The result is

$$\frac{\partial}{\partial t}\mathbf{P}^w = -\rho g \nabla . \int d\mathbf{k}\, \frac{\mathbf{l}\mathbf{v}_g}{c}\, F + \rho g \int d\mathbf{k}\, \frac{\mathbf{l}}{c} \left(S_{in} + S_{nl} + S_{diss} + S_{bot}\right). \qquad (14)$$

Elimination of the wave momentum from (12) gives the following evolution equation for the ocean circulation velocity $\mathbf{U}_c$

$$\left(\frac{\partial}{\partial t}+\mathbf{U}_c.\nabla_h\right)\mathbf{U}_c+g\nabla_h\zeta+\frac{1}{\rho}\nabla_h p_a=(\mathbf{U}_c+\mathbf{U}_{surf})\times\mathbf{f}+\frac{\tau_{oc,a}-\tau_{oc,b}}{\rho h}-\frac{1}{\rho h}\nabla_h.\mathsf{T}, \tag{15}$$

and it is straightforward to rewrite the continuity equation:

$$\frac{\partial}{\partial t}\zeta+\nabla_h.(h\mathbf{U}_c)=-\nabla_h.\left(h\mathbf{U}_{surf}\right). \tag{16}$$

Now, the Stokes-Coriolis force shows up as the $\mathbf{U}_{surf}\times\mathbf{f}$ term. The reason that it shows up in the equation for the total ocean circulation is that wave momentum evolution is not affected by rotation on earth.

Secondly, the stress tensor simplifies to

$$T_{ij}=\rho g\int d\mathbf{k}\left(\frac{v_g}{c}-\frac{1}{2}\right)\delta_{ij}F(\mathbf{k}). \tag{17}$$

Finally, the surface stress and the bottom stress are modified accordingly. For example, the surface stress felt by the mean circulation is the total stress minus the net stress going into the waves, or,

$$\tau_{oc,a} = \tau_a - \rho g \int d\mathbf{k}\, \mathbf{l}\,(S_{in} + S_{nl} + S_{diss})/c, \tag{18}$$

and the bottom stress becomes

$$\tau_{oc,b} = \tau_b + \rho g \int d\mathbf{k}\, \mathbf{l} S_{bot}/c. \tag{19}$$

THE SURFACE DRIFT PROFILE

Consider first a single gravity wave at the interface of air and water. Suppose the surface elevation is given by

$$\eta = a\cos\theta,\ \theta = kx - \omega t, \tag{20}$$

hence, we take a wave with amplitude a, wavenumber k and angular frequency ω which is propagating to the right. The question now is what is in a Eulerian frame the mean momentum as function of height z. Clearly, since this is a periodic wave there is no mean momentum below $z = -a$ or above $z = a$, hence only mean momentum for $|z| < a$ will be found.

The mean momentum at height z is

$$P = \frac{1}{2\pi}\int_0^{2\pi} d\theta\ \rho u = \frac{\rho_w}{2\pi}\int_{-\theta_0}^{\theta_0} d\theta\ u_w + \frac{\rho_a}{2\pi}\int_{\theta_0}^{2\pi-\theta_0} d\theta\ u_a, \tag{21}$$

where θ_0 follows from

$$z = a\cos\theta,$$

hence

$$\theta_0 = \arccos(z/a). \tag{22}$$

Here, the subscripts a and w refer to air and water, respectively. For simplicity we assume potential flow $u = \partial\phi/\partial x = k\partial\phi/\partial\theta$ where

$$\begin{aligned} \phi_w &= +cae^{kz}\sin\theta \\ \phi_a &= -cae^{-kz}\sin\theta \end{aligned} \tag{23}$$

with $c = \omega/k$ is the wave speed. (Note that across the interface the u component of the velocity jumps, while the vertical component is continuous), and the mean momentum becomes

$$P = \frac{\rho_a + \rho_w}{\pi} k\phi(\theta_0),$$

or,

$$P = \frac{\omega a}{\pi}(\rho_a + \rho_w)\sin\theta_0, \tag{24}$$

where we ignore the $\exp(-k|z|)$ factor because we take weakly nonlinear waves, hence, $ka \ll 1$.

Now, $\sin\theta_0 = \sqrt{1-\cos^2\theta_0} = \sqrt{1-(z/a)^2}$, and therefore the height dependence of the mean momentum follows from

$$P = \frac{1}{2}(\rho_a + \rho_w)\omega a^2 d(z,a), \tag{25}$$

where for small amplitude a the function $d(z,a)$ is highly localized around $z = 0$,

$$d(z,a) = \frac{2}{\pi a}\sqrt{1-\left(\frac{z}{a}\right)^2}, \tag{26}$$

which is normalized to 1, ie. $\int dz\, d(z,a) = 1$. In particular in the limit $a \to 0$ the function $d(z,a)$ behaves like a δ-function and hence the surface drift becomes a surface jet:

$$\lim_{a\to 0} P \to \frac{1}{2}(\rho_a + \rho_w)\omega a^2 \delta(z), \tag{27}$$

and one would expect that such a highly singular jet, which has the same momentum as the Stokes drift, should play a role in the mean momentum equations.

SURFACE DRIFT and OCEAN CIRCULATION

Linear Theory

Starting point are the equations for an adiabatic fluid on a rotating earth, and we consider phenomena with a speed much smaller than the sound speed, hence

$$\begin{aligned}\nabla.\mathbf{u} &= 0,\\ \frac{\partial}{\partial t}\rho\mathbf{u}+\nabla.\rho\mathbf{u}\mathbf{u} &= -\nabla p+\rho\mathbf{g}+\rho\mathbf{u}\times\mathbf{f},\\ \left(\frac{\partial}{\partial t}+\mathbf{u}.\nabla\right)\rho &= 0.\end{aligned} \tag{28}$$

The equilibrium is given by

$$\begin{aligned}\mathbf{u}_0 = 0,\ \mathbf{g} = -g\hat{e}_z, \rho_0 = \rho_0(z),\\ p_0 = p(z) = -g\int dz\,\rho_0(z)\end{aligned} \tag{29}$$

First we shall apply linear theory, which enables us to obtain the necessary fluxes in the mean flow equations. The resulting fluxes will induce a mean flow, but we shall assume that the waves are much faster than the mean flow. In other words, mean flow effects on the waves can be ignored.

Consider a plane wave propagating in the x-direction so there is no y-dependence. The perturbations are assumed to have the form

$$(\delta\rho, \delta u, \delta v, \delta w, \delta p) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{p})\, e^{i\theta} + c.c,\ \theta = kx - \omega t, \tag{30}$$

where the amplitudes are still functions of height z. Linearizing Eq. (28) around the equilibrium (29) then gives

$$\begin{aligned}
ik\hat{u} + \hat{w}' &= 0, \\
i\omega\hat{u} &= ik\hat{p}/\rho_0 - f\hat{v}, \\
i\omega\hat{v} &= f\hat{u}, \\
i\omega\hat{w} &= g\hat{\rho}/\rho_0 + \hat{p}'/\rho_0, \\
i\omega\hat{\rho} &= \hat{w}\rho_0',
\end{aligned} \tag{31}$$

where a prime denotes differentiation with respect to z.

Combining the first and third equation of (31) we have

$$\hat{v} = \frac{f\hat{w}'}{\omega k},\ \hat{u} = -\frac{\hat{w}'}{ik} \tag{32}$$

hence cross velocity and vertical velocity are in phase, giving a non-zero flux $\rho_0\langle\delta v\delta w\rangle$ which produces a force orthogonal to the wave propagation direction.

The density perturbation becomes

$$\hat{\rho} = \rho_0'\frac{\hat{w}}{i\omega} \tag{33}$$

while from the second equation of (31) the pressure perturbation becomes

$$\hat{p} = i\frac{\omega}{k^2}\rho_0 D\hat{w}',\ D = 1-(f/\omega)^2. \tag{34}$$

Note that in practice for surface gravity waves $f \ll \omega$, hence $D \to 1$.

Finally, eliminating pressure and density perturbation from the fourth equation of (31) we arrive at the Sturm-Liouville type of differential equation

$$\frac{d}{dz}\left(\rho_0 \frac{d}{dz}\hat{w}\right) = \frac{\kappa^2 \hat{w}}{\omega^2}\left(g\rho_0' + \omega^2 \rho_0\right), \tag{35}$$

where $\kappa^2 = k^2/D$, and the boundary conditions are the vanishing of the vertical velocity at infinity:

$$\hat{w} \rightarrow 0 \text{ for } |z| \rightarrow \infty. \tag{36}$$

We do not use the so-called Boussinesq approximation (ignore all density variations except in combination with acceleration of gravity) because, in particular for surface gravity waves the density gradient is large.

The dispersion relation is obtained from (35) by multiplication with $\hat{w}^*$ and integration of the result from $z \to -\infty$ to $z \to +\infty$. Partial integration of the LHS, and making use of the boundary condition for $\hat{w}$ then gives

$$\omega^2 \int_{-\infty}^{\infty} dz\, \rho_0 \left\{ |\hat{w}'|^2 + \kappa^2 |\hat{w}|^2 \right\} = -g\kappa^2 \int_{-\infty}^{\infty} dz\, \rho_0' |\hat{w}|^2. \tag{37}$$

Considering only the case of high-frequency waves, such as surface gravity waves are, we ignore f with respect to ω in κ. Then, if the density profile is stable ($\rho_0' < 0$) we have real solutions for angular frequency ω, a result which is well-known.

Quasi-linear fluxes

Next, we will derive some general expressions for the relevant fluxes. First, consider the stress along the wave direction,

$$\tau_{uw} = -\rho_0 \langle \delta u \delta w \rangle = -\rho_0 \left(\hat{u}\hat{w}^* + c.c \right) \tag{38}$$

Using (32) one finds

$$\tau_{uw} = \frac{\mathscr{W}}{k} \tag{39}$$

where $\mathscr{W}$ is the Wronskian of the differential equation (35),

$$\mathscr{W} = -i\rho_0 \left(\hat{w}' \hat{w}^* - \hat{w}^{*\prime} \hat{w} \right). \tag{40}$$

If there are no critical layers the Wronskian is constant. This follows immediately by differentiating $\mathscr{W}$ with respect to z, and using (35)

$$\frac{d}{dz}\mathscr{W} = -i\kappa^2 |\hat{w}|^2 \left(\frac{g\rho_0'}{\omega^2} + \rho_0 \right) + c.c, \tag{41}$$

and this vanishes because for real ω the term in brackets is real. Hence, τ_{uw} is constant and since $\hat{w}$ vanishes for large heights we conclude that τ_{uw} vanishes.

Remark that the vanishing of τ_{uw} depends on ω being real. In unsteady circumstances there will be a finite stress. Unsteadiness can be mimicked by introduction of a slight damping in the system of equations. In Eqns. (28) we replace

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \varepsilon \tag{42}$$

everywhere, where ε is a small damping rate. This effectively means that angular frequency ω is replaced by $\omega - i\varepsilon$. For complex frequency the right-hand side of (41) does not vanish. As a consequence one finds as force along the wave

$$\frac{\partial}{\partial z}\tau_{uw} = -2\frac{gk}{\omega^2}\rho_0' \frac{1}{\omega}\frac{\partial}{\partial t}|\hat{w}|^2, \tag{43}$$

hence the force is proportional to the time derivative of the vertical velocity (cf. Andrews and McIntyre, 1976). Note that for surface gravity waves, when the density shows a jump at $z = 0$, the above force is singular.

The next flux of interest is

$$\tau_{vw} = -\rho_0 \langle \delta v \delta w \rangle = -\rho_0 (\hat{v}^* \hat{w} + c.c) \tag{44}$$

Using (32) this may be written as

$$\tau_{vw} = -\frac{f}{\omega k} \left(\rho_0 \hat{w}^* \hat{w}' + c.c. \right). \tag{45}$$

Differentiating the stress with respect to z and making use of the differential equation for $\hat{w}$ gives the important relation

$$\frac{\partial}{\partial z} \tau_{vw} = -\frac{f}{\omega k} \left\{ \rho_0 \left[\kappa^2 |\hat{w}|^2 + |\hat{w}'|^2 \right] + \frac{g\kappa^2}{\omega^2} \rho_0' |\hat{w}|^2 \right\} + c.c.\,. \tag{46}$$

The force given in Eq. (46) consists of two parts. The first part is given by the term in square brackets and is a regular function of height, because, although $\hat{w}'$ may show a jump at the air-sea interface, $|\hat{w}'|^2$ is continuous. For the water wave problem this will give rise to the $\mathbf{u}_s \times \mathbf{f}$ - force. The second part is proportional to the density gradient. This part is, however, of a special nature because for the air-water problem ρ_0 shows a jump, hence $\rho_0' \sim \delta(z)$. Therefore, there is a very important contribution of the force very close to the surface. It is important to retain this $\delta(z)$-force, however, because of momentum conservation.

Surface Gravity Waves

Let us now consider the special case of surface gravity waves by taking as density profile

$$\rho_0 = \begin{cases} \rho_a, \ z > 0, \\ \\ \rho_w, \ z < 0. \end{cases} \tag{47}$$

where ρ_a and ρ_w are constants. The density ratio $\varepsilon = \rho_a/\rho_w$ is assumed to be small.

The relevant wave-induced forces (43) and (46) can now immediately be evaluated. One finds

$$\begin{aligned} \frac{\partial}{\partial z}\tau_{uw} &= \frac{\partial}{\partial t}\rho_0 u_{surf}, \\ \frac{\partial}{\partial z}\tau_{vw} &= -f\rho_0\left(u_{stokes} - u_{surf}\right) \end{aligned} \tag{48}$$

where

$$\rho_0 u_{surf} = 2\left(\rho_a + \rho_w\right)\delta(z)\omega|\hat{\eta}|^2, \ u_{stokes} = 4\omega k|\hat{\eta}|^2 e^{-2k|z|}. \tag{49}$$

With the identification $2\hat{\eta} \rightarrow a$ the surface momentum in (49) is found to be identical to the mean momentum (27) from the simple considerations given previously.

It is of interest to plot the cross stress $\tau_{vw} = 2\rho_0 f \omega |\hat{\eta}|^2 e^{-2k|z|} sign(z)$ as function of z.

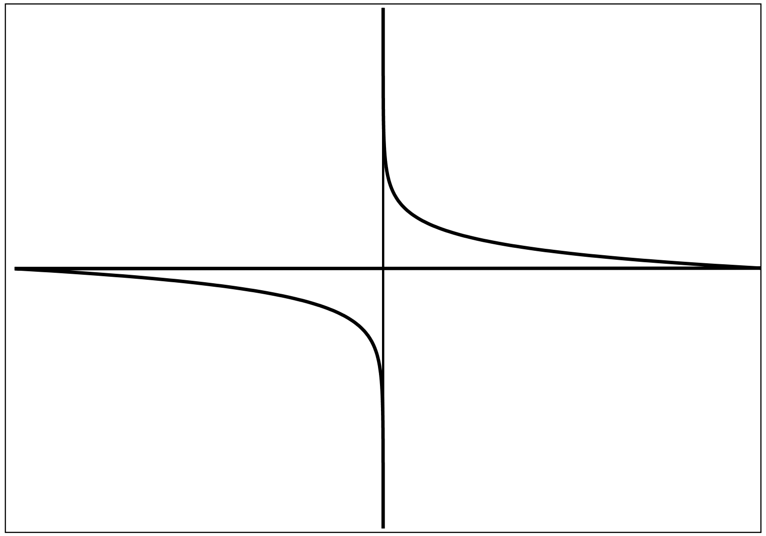

Figure 1: Profile of cross-stress τ_{vw}. For display purposes air and water density are equal.

Thus strictly speaking there is a Stokes-Coriolis force both in air and water!

Many waves

So far we considered a single wave but the generalization of these results to the case of many waves with random phase is immediate. Simply replace $|\hat{\eta}|^2$ by $F(\omega)/2$ with $F(\omega)$ the wave spectrum and integrate over ω. The surface and the Stokes drift become

$$\begin{aligned} \rho_0 u_{surf} &= (\rho_a + \rho_w)\delta(z) \int_0^\infty d\omega\ \omega F(\omega), \\ \rho_0 u_{stokes} &= \frac{2}{g}\rho_0 \int_0^\infty d\omega\ \omega^3 F(\omega) e^{-2k|z|},\ k = \omega^2/g. \end{aligned} \tag{50}$$

Furthermore, the variance in the vertical velocity, relevant for the wave-induced dynamic pressure, can be related to the wave spectrum as well as

$$\langle \delta w^2 \rangle = \int_0^\infty d\omega\ \omega^2 F(\omega) e^{-2k|z|}. \tag{51}$$

Consequences for ocean circulation

Generalizing to the case of arbitrary wave propagation direction, and adding the effects of turbulent momentum transport through the divergence of a turbulent stress τ_{turb} one finds as total circulation equation

$$\frac{\partial}{\partial t}\rho\left(\mathbf{u}-\mathbf{u}_{surf}\right) = -\nabla\langle p\rangle + \rho\left(\mathbf{u}-\mathbf{u}_{surf}\right)\times\mathbf{f} + \rho\mathbf{u}_{stokes}\times\mathbf{f} + \frac{\partial}{\partial z}\tau_{turb}, \tag{52}$$

where

$$\langle p\rangle = -\rho_0\langle\delta w^2\rangle - g\int dz\,\rho_0. \tag{53}$$

Consistent with our discussion on the total column budget it is immediately evident from Eq. (52) that there is a natural distinction between the total current $\mathbf{u}$ and the surface drift $\mathbf{u}_{surf}$. Furthermore, it seems natural to introduce the ocean circulation velocity $\mathbf{u}_c = \mathbf{u} - \mathbf{u}_{surf}$, in complete agreement with the practice that has been developed for the total momentum budget, cf. Eq. 10.

The Stokes-Coriolis force seems to be important for the ocean circulation. Defining the transport as

$$\mathbf{T} = \int_{-\infty}^{0} dz\, \rho \mathbf{u}_c \tag{54}$$

then in the steady state a result is found which deviates from the classical Ekman spiral:

$$\mathbf{T} = \frac{\tau_0 \times \mathbf{f}}{f^2} - \int dz\, \rho \mathbf{u}_{stokes}, \tag{55}$$

Using the well-known expression of the Stokes drift in terms of the wave spectrum, the final result is

$$\mathbf{T} = \frac{\tau_0 \times \mathbf{f}}{f^2} - \rho_0 \int d\omega\, \omega F(\omega), \tag{56}$$

In the extra-tropics the Stokes-Coriolis effect can be estimated to be 30% of the total transport, therefore this effect should be taken into account in the modelling of the ocean circulation. Caveats are the steady state assumption and stratification effects in the ocean which reduce turbulent transport (compare Polton *et al* with Ardhuin c.s.).

Using a constant eddy viscosity model it is straightforward to show the impact of the Stokes-Coriolis force on the Ekman Spiral:

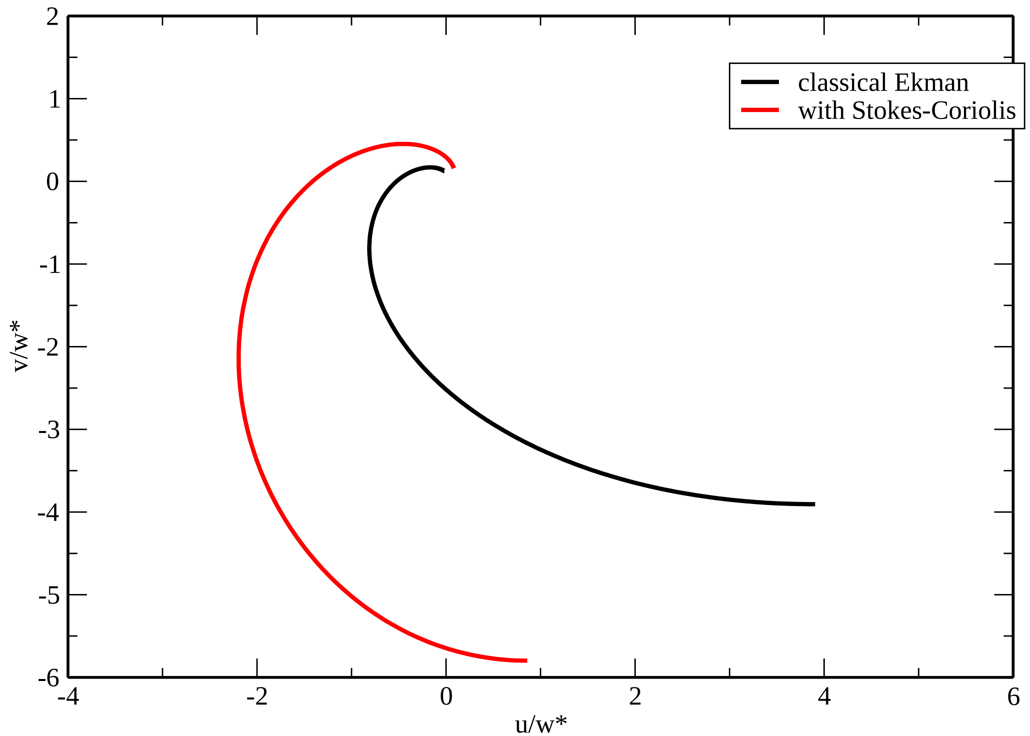

Figure 2: Ekman spiral for case of constant eddy viscosity. A comparison is made between the classical Ekman spiral and the case of Stokes-Coriolis forcing by a single wave train.

Evidence from simulations with NEMO

At ECMWF we use the NEMO ocean model. All experiments reported here are with a spatial resolution of 1°and 75 layers in the vertical with a top-layer of 1 m. Forcing is provided by ERA-interim and the period is more than 20 years. Fluxes from bulk formula!!

The Stokes drift induced by the ocean waves combined with the Coriolis effect gives rise to an additional force which is to the right (left) of the wind direction in the Northern (Southern) Hemisphere. This has impact on the instantaneous currents and also on the average SST in Winter (DJF) and Summer (JJA).

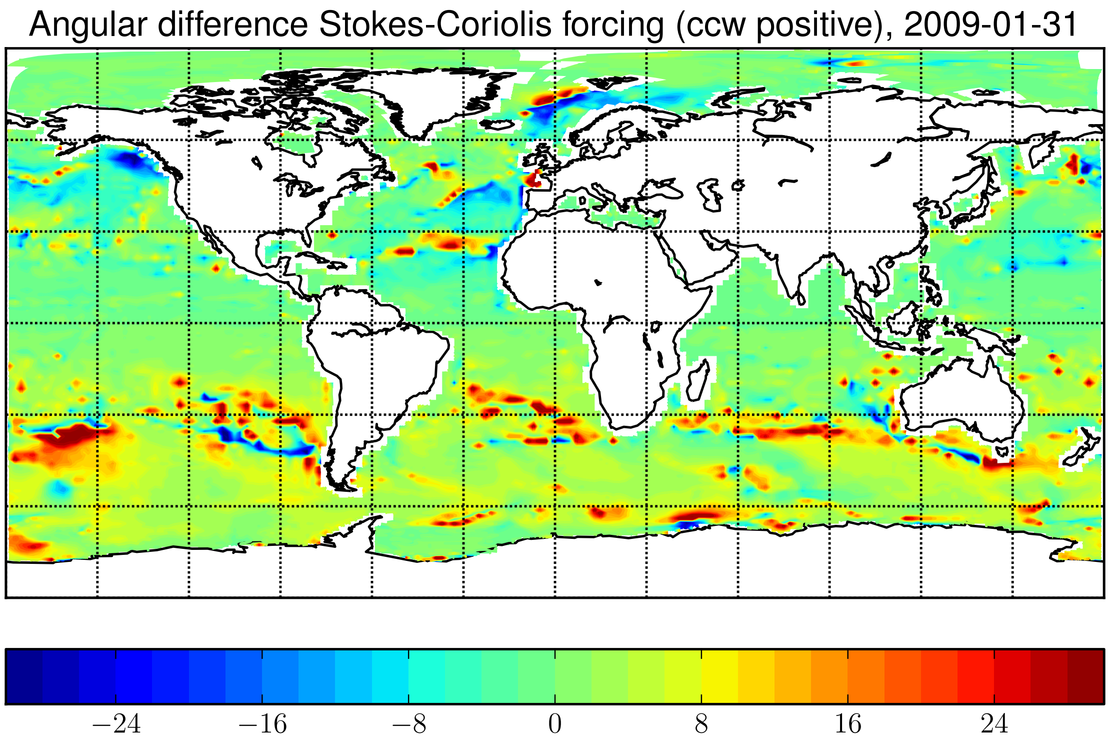

Figure 3: Impact of Stokes-Coriolis forcing on current direction.

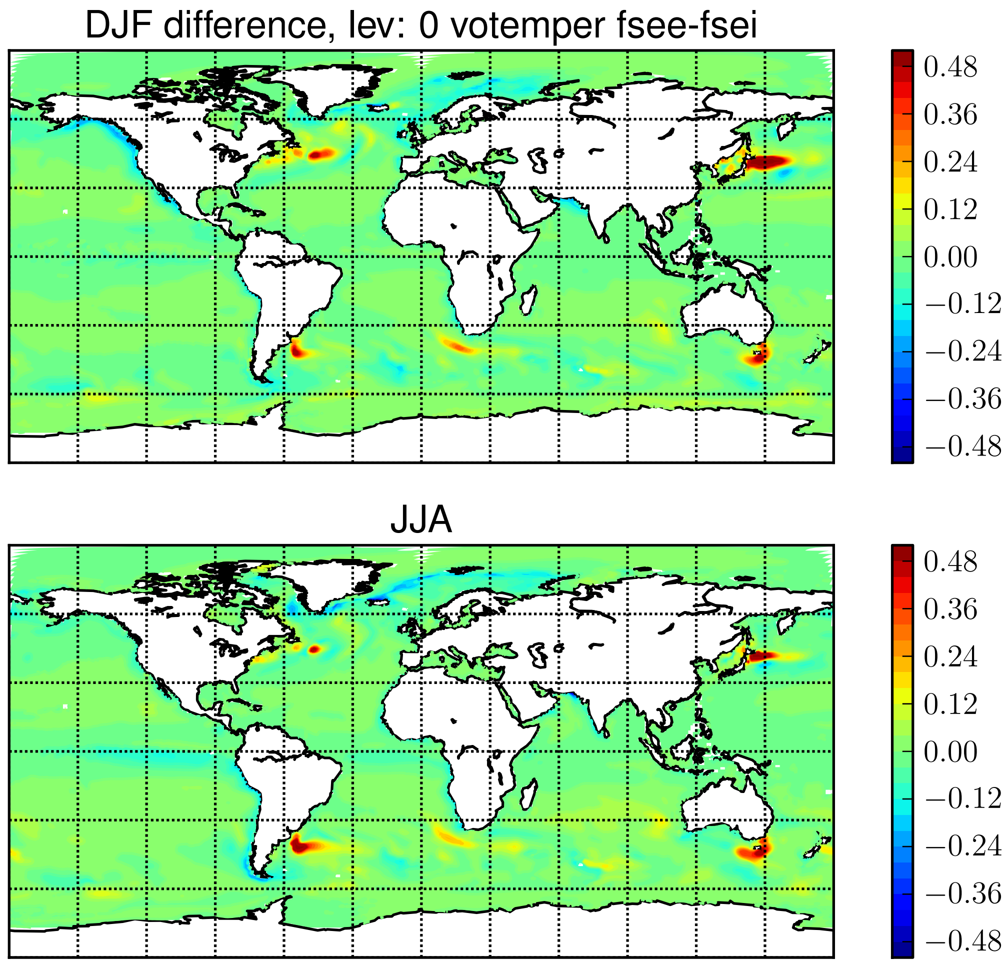

Figure 4: Impact of Stokes-Coriolis forcing on SST averaged over a 20 year period.

WAVE BREAKING and UPPER OCEAN MIXING

In the past 15 years observational evidence has been presented about the role of wave breaking and Langmuir turbulence in the upper ocean mixing.

Because wave breaking generates turbulence near the surface, in a layer of the order of the wave height H_S, the turbulent velocity is enhanced by a factor of 2-3, while, in agreement with observations there is an enhanced turbulent dissipation. This deviates from the ’law-of-the-wall’.

The turbulence modelling is based on an extension of the **Mellor-Yamada scheme** with sea state effects. Combined with a proper modelling of buoyancy effects a realistic simulation of the diurnal cycle may be obtained. Here, the energy flux from waves to ocean column follows from the dissipation term in the energy balance equation:

$$\Phi_{oc} = -\rho_w g \int d\omega d\theta \, S_{ds}.$$

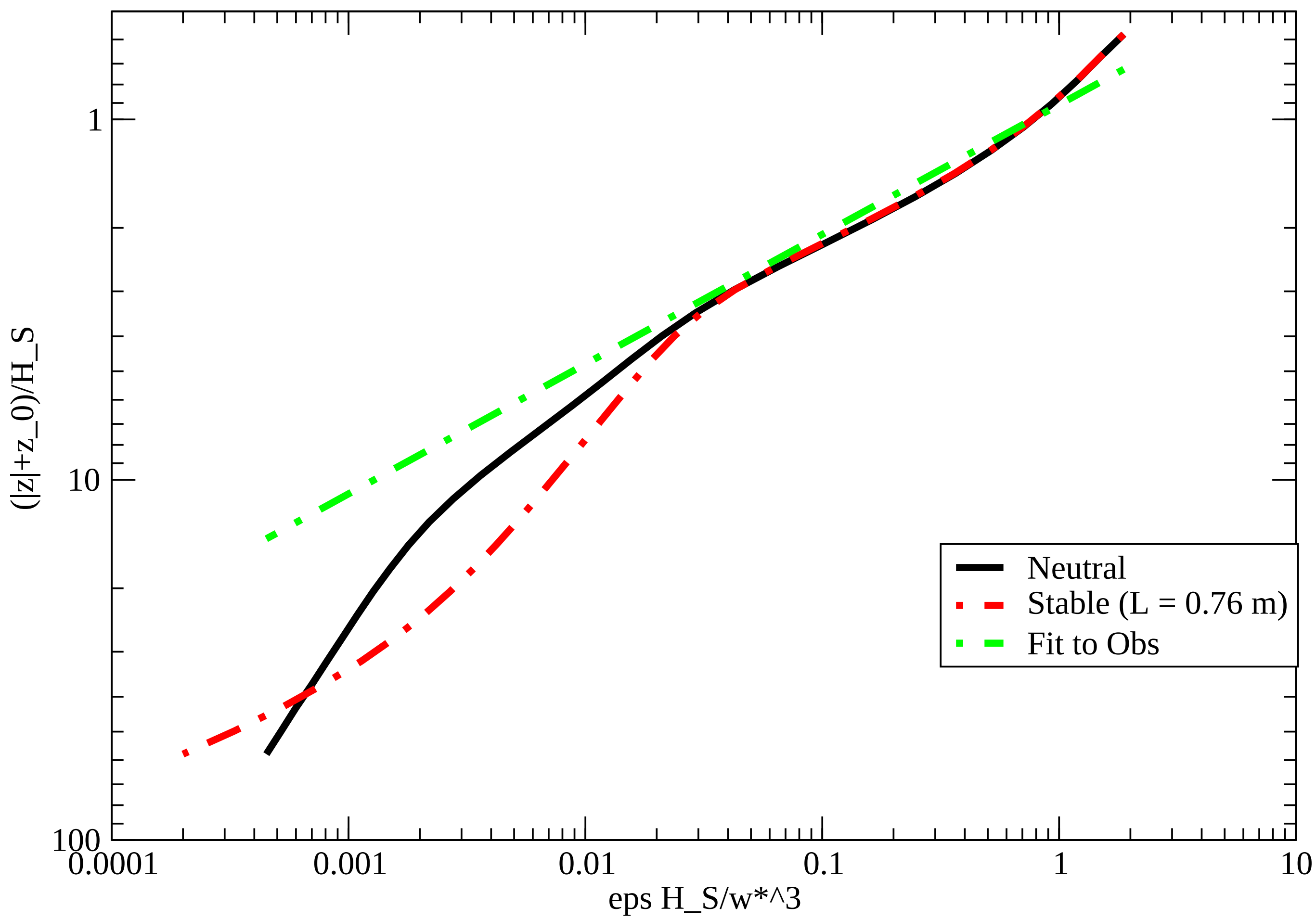

Figure 5: Dimensionless dissipation $\varepsilon_* = \varepsilon H_S / \Phi_{oc}$ versus $(z + z_0)/H_S$

TKE EQUATION

The enhanced turbulent dissipation can be described in the context of the turbulent kinetic energy (TKE) equation. If effects of advection are ignored, the TKE equation describes the rate of change of turbulent kinetic energy e due to processes such as shear production (including the shear in the Stokes drift), damping by buoyancy, vertical transport of pressure and TKE, and turbulent dissipation ε. It reads

$$\frac{\partial e}{\partial t} = \nu_m S^2 + \nu_m S \frac{\partial U_S}{\partial z} - \nu_h N^2 + \frac{1}{\rho_w} \frac{\partial}{\partial z} (\overline{\delta p \delta w}) + \frac{\partial}{\partial z} (\overline{e \delta w}) - \varepsilon,$$

where $e = q^2/2$, with q the turbulent velocity, $S = \partial U / \partial z$ and $N^2 = g \rho_0^{-1} \partial \rho / \partial z$, with N the Brunt-Väisälä frequency, ρ_w is the water density, δp and δw are the pressure and vertical velocity fluctuations and the over-bar denotes an average taken over a time scale that removes linear turbulent fluctuations. The eddy viscosities for momentum and heat are denoted by ν_m and ν_h. I use $\nu_m = l(z) q(z) S_M$ where $l(z)$ is the mixing length and S_M depends on stratification.

The turbulent production of Langmuir circulation is modelled by the second term on the right-hand side of the TKE equation which represents works against the shear in the Stokes drift. Here U_S is the magnitude of the Stokes drift for a general wave spectrum $F(\omega)$,

$$U_S = \frac{2}{g}\int_0^\infty d\omega\, \omega^3 F(\omega) e^{-2k|z|},\ k = \omega^2/g.$$

Although in principle the depth dependence of the Stokes drift is known it still is a fairly elaborate expression through the above integral. In the final result we will use the approximate expression

$$U_S = U_S(0)e^{-2k_S|z|},$$

where $U_S(0)$ is the value of the Stokes drift at the surface and k_S is an appropiately chosen wavenumber scale.

The dissipation term is taken to be proportional to the cube of the turbulent velocity divided by the mixing length $l = \kappa|z|$,

$$\varepsilon = \frac{q^3}{Bl},$$

Here, B is another dimensionless constant.

The pressure transport term can be determined by explicitely modelling the energy transport caused by wave dissipation. The correlation between pressure fluctuation and vertical velocity fluctuation at the surface is

$$I_w(0) = +\frac{1}{\rho_w}\overline{\delta p \delta w}(z=0) = \frac{\Phi_{oc}}{\rho_w} = -g\int_0^\infty S_{diss}(\mathbf{k})d\mathbf{k} = m\frac{\rho_a}{\rho_w}u_*^3 = m\frac{\rho_w^{1/2}}{\rho_a^{1/2}}w_*^3,$$

where u_* is the air friction velocity while w_* is the water friction velocity which follows from $\rho_a u_*^2 = \rho_w w_*^2$.

and the main problem is how to model the depth dependence of $\overline{\delta p \delta w}$. Assume depth scale is controlled by significant wave height H_S:

$$I_w(z) = +\frac{1}{\rho_w}\overline{\delta p \delta w} = I_w(0) \times \hat{I}_w,\ \hat{I}_w = e^{-|z|/z_0}$$

where the depth scale $z_0 \sim H_S$ will play the role of a roughness length. Thus, the TKE equation becomes

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left(lqS_q\frac{\partial e}{\partial z}\right) + \frac{\partial I_w(z)}{\partial z} + \nu_m S^2 + w_*^2\frac{\partial U_S}{\partial z} - \nu_h N^2 - \frac{q^3}{Bl(z)}.$$

At the surface there is no direct conversion of mechanical energy to turbulent energy and therefore the turbulent energy flux is assumed to vanish. Hence the boundary conditions become

$$lqS_q\frac{\partial e}{\partial z} = 0 \qquad \text{for} \qquad z = 0,$$

$$\frac{\partial e}{\partial z} = 0 \qquad \text{for} \qquad z \to \infty.$$

STEADY STATE PROPERTIES

NEUTRALLY STABLE

The properties of the steady state version of the TKE equation were studied extensively. Without presenting any of the details, for neutral stratification the following **'1/3'-rule** is found. Introducing the dimensionless turbulent velocity $Q = (S_M/B)^{1/4} \times q/w_*$ the approximate solution of the TKE equation becomes

$$Q^3 \approx 1 + m\sqrt{\frac{\rho_w}{\rho_a}}\kappa|z|\frac{d\hat{I}_w}{dz} + La^{-2}\kappa|z|\frac{d\hat{U}_S}{dz},$$

where $La = (w_*/U_S(0))^{1/2}$ is the turbulent Langmuir number. So, in terms of Q^3 there is a **superposition principle**, i.e. contributions due to wave dissipation and Langmuir turbulence may be added to the shear production term.

The next graph shows the contributions of wave dissipation and Langmuir turbulence to the turbulent velocity

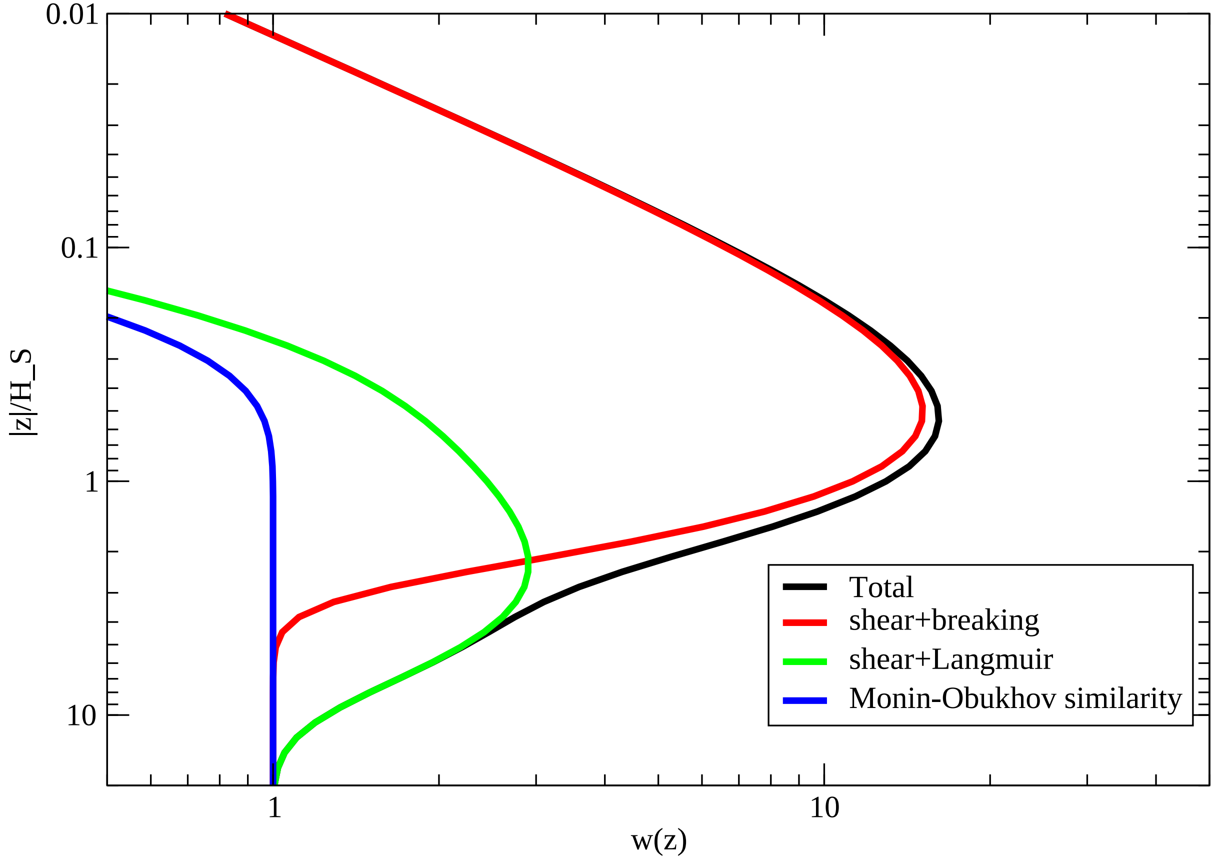

Figure 6: Profile of Q^3 according to the local approximation in the ocean column near the surface. The contributions by wave dissipation (red line) and Langmuir turbulence (green line) are shown as well. Finally, the *w*-profile according to Monin-Obukhov similarity, which is basically the balance between shear production and dissipation, is shown as the blue line.

IMPACT ON DIURNAL CYCLE

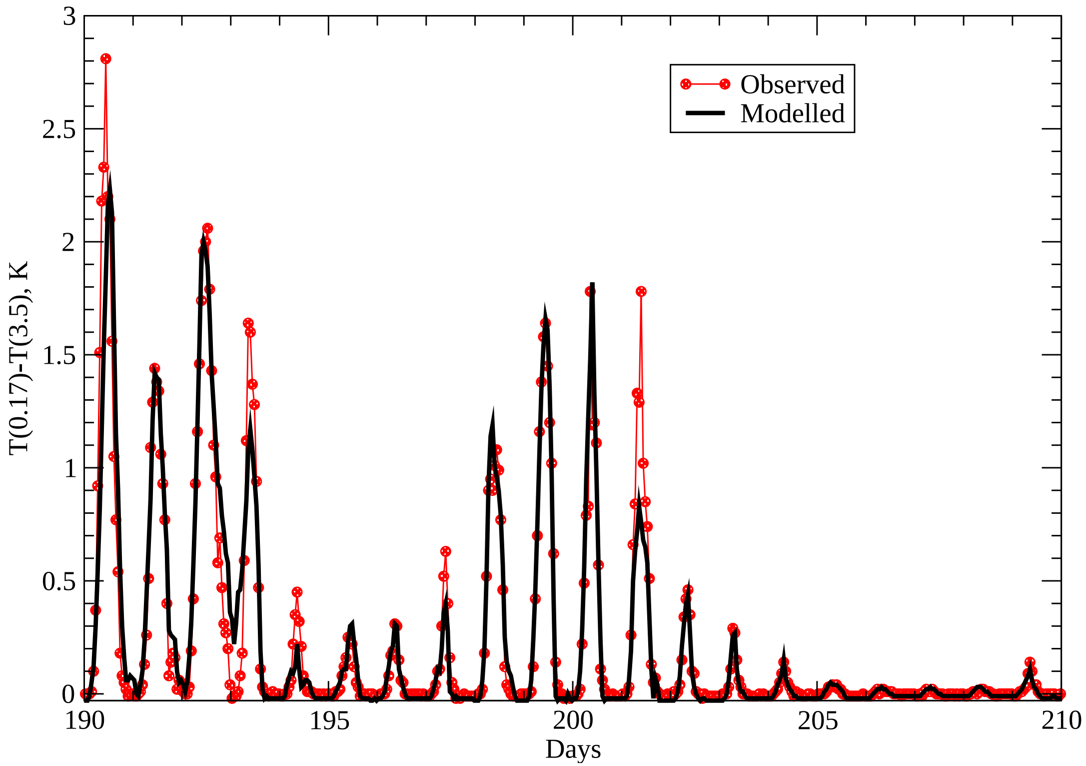

Figure 7: Observed and simulated ocean temperature $\Delta T = T(0.17) - T(3.5)$ at 15^o30' N, 61^o30' E in the Arabian Sea for 20 days from the 23rd of April.

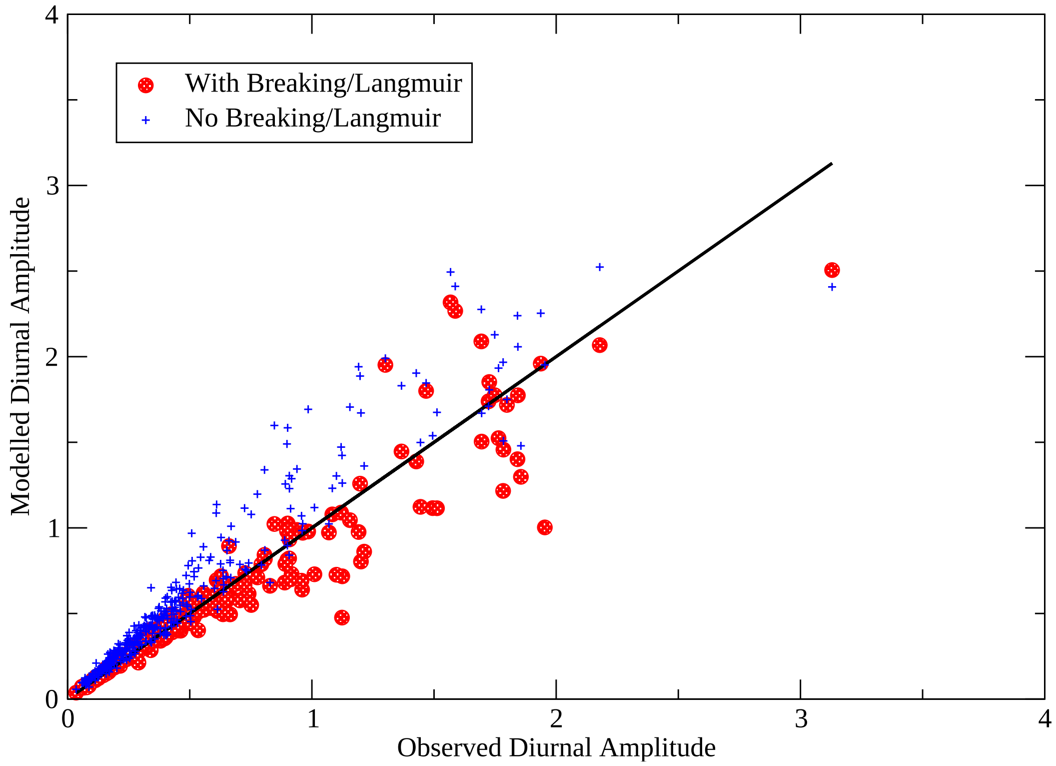

Figure 8: Comparison of simulated and observed diurnal amplitude at $15^o30'$ N, $61^o30'$ E in the Arabian Sea for the one year period starting from 16th of October 1994.

IMPACT ON MEAN SST FIELD

The insights gained during the diurnal cycle work have been used in further developing the NEMO ocean model. The ocean model is forced by the momentum flux to the ocean column, while the mixing due to wave breaking now explicitely depends on the dissipation produced by the WAM model. The Stokes drift is determined as well. This allows for explicitely taking into account the effects of Langmuir turbulence and the Stokes-Coriolis force.

Specifically the momentum flux to the ocean is given by

$$\tau_{oc} = \tau_a - \rho_w g \int_0^{2\pi} \int_0^{\infty} d\omega d\theta \, \mathbf{k} \left(S_{in} + S_{diss} \right) / \omega, \tag{57}$$

while the energy flux to the ocean reads

$$\Phi_{oc} = -\rho_w g \int_0^{2\pi} \int_0^{\infty} d\omega d\theta \, S_{diss}, \tag{58}$$

Monthly means of these quantities are shown in the next two figures.

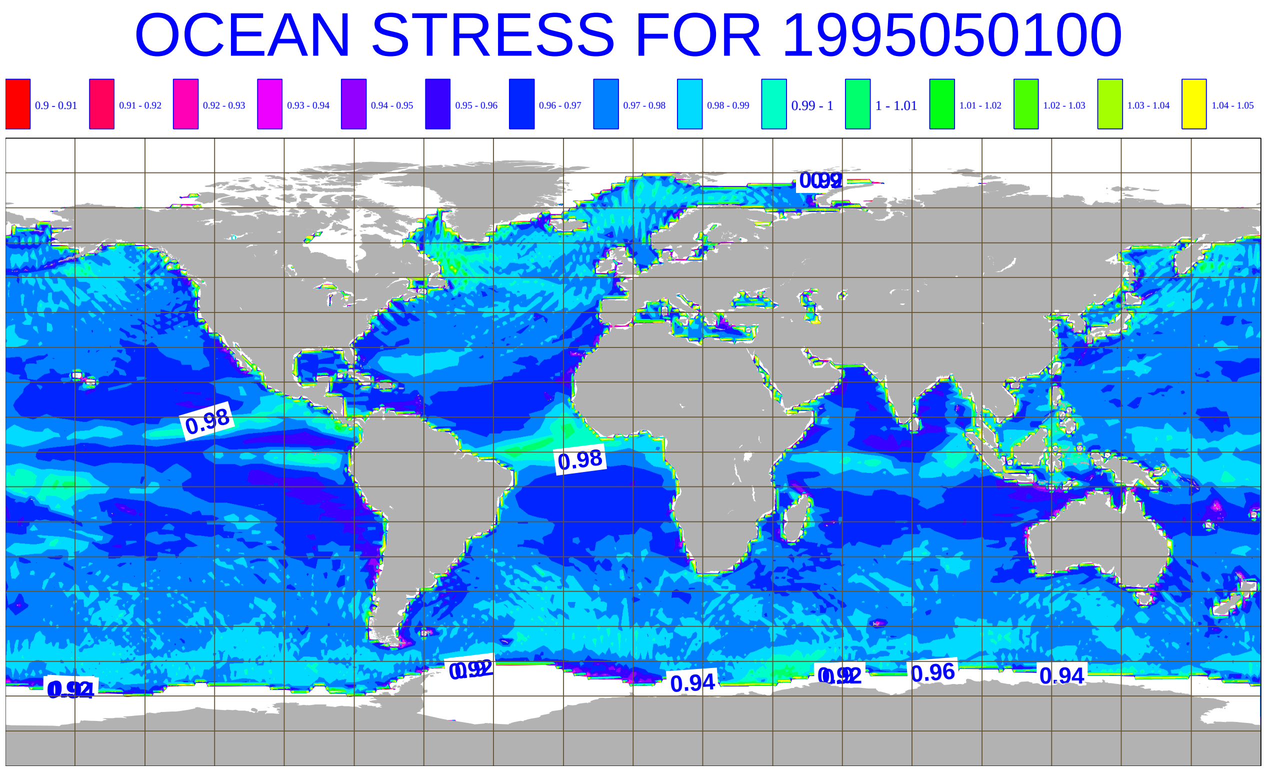

Figure 9: Monthly mean of momentum flux into the ocean, normalized with the monthly mean of the atmospheric stress.

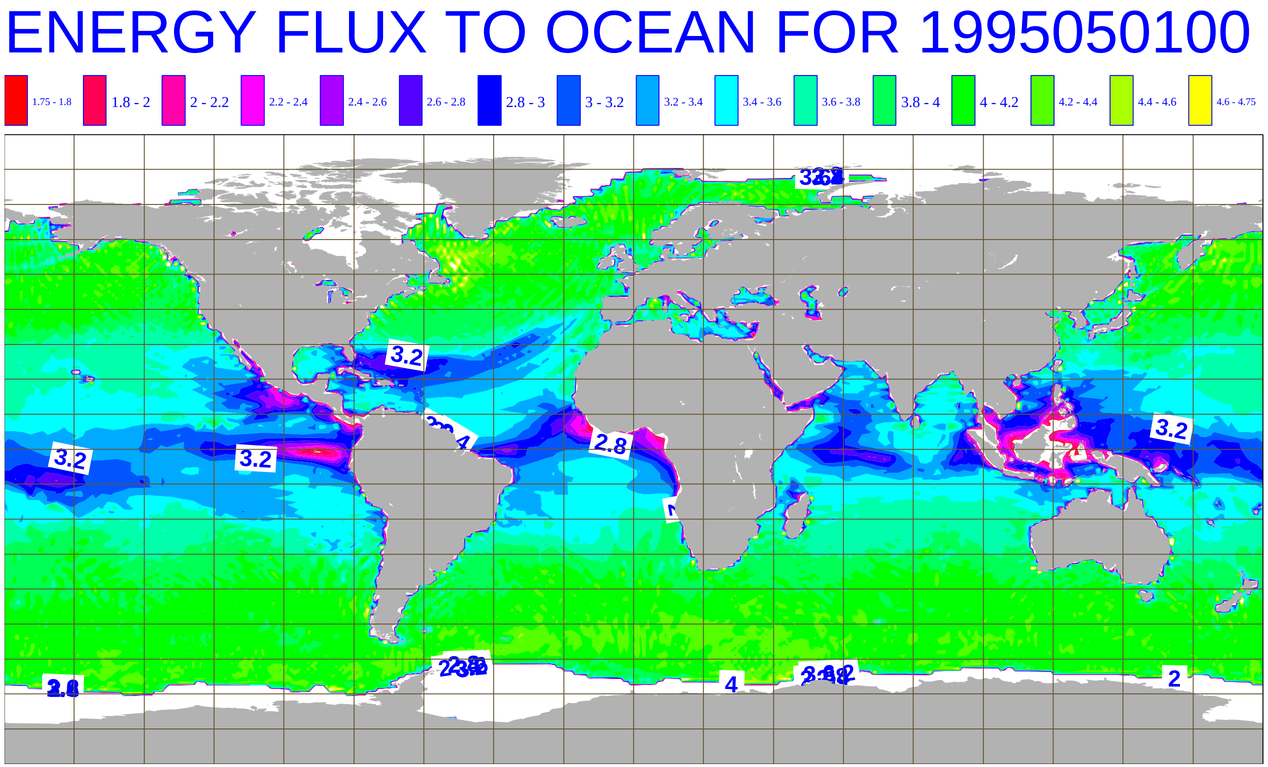

Figure 10: Monthly mean of energy flux into the ocean, normalized with the monthly mean of $\rho_a u_*^3$.

Impact continue

We compare the average of the SST field produced by the NEMO model with explicit sea state effects (hence the dimensionless dissipation flux m is determined by the WAM model) with a version of NEMO where m is given a constant value, appropriate for old wind sea.

Considerable differences in SST are found.

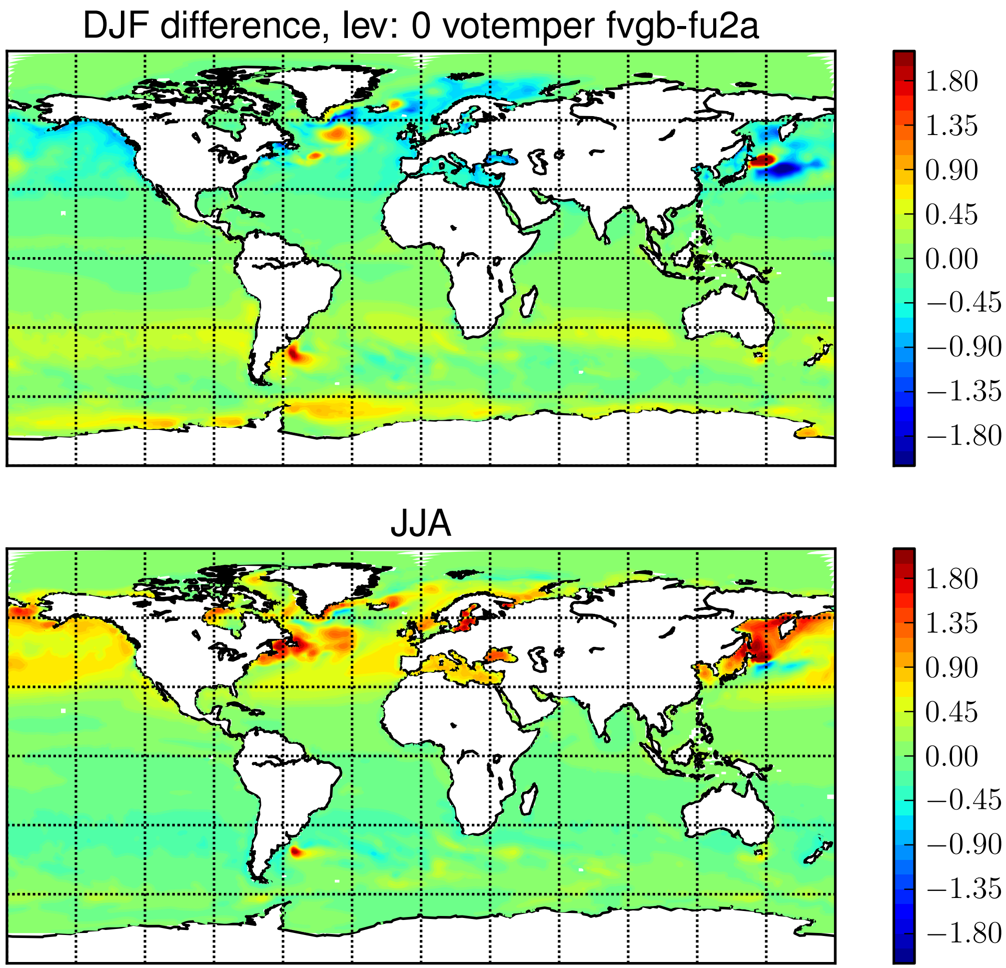

Figure 11: Combined impact on SST averaged over a 20 year period.

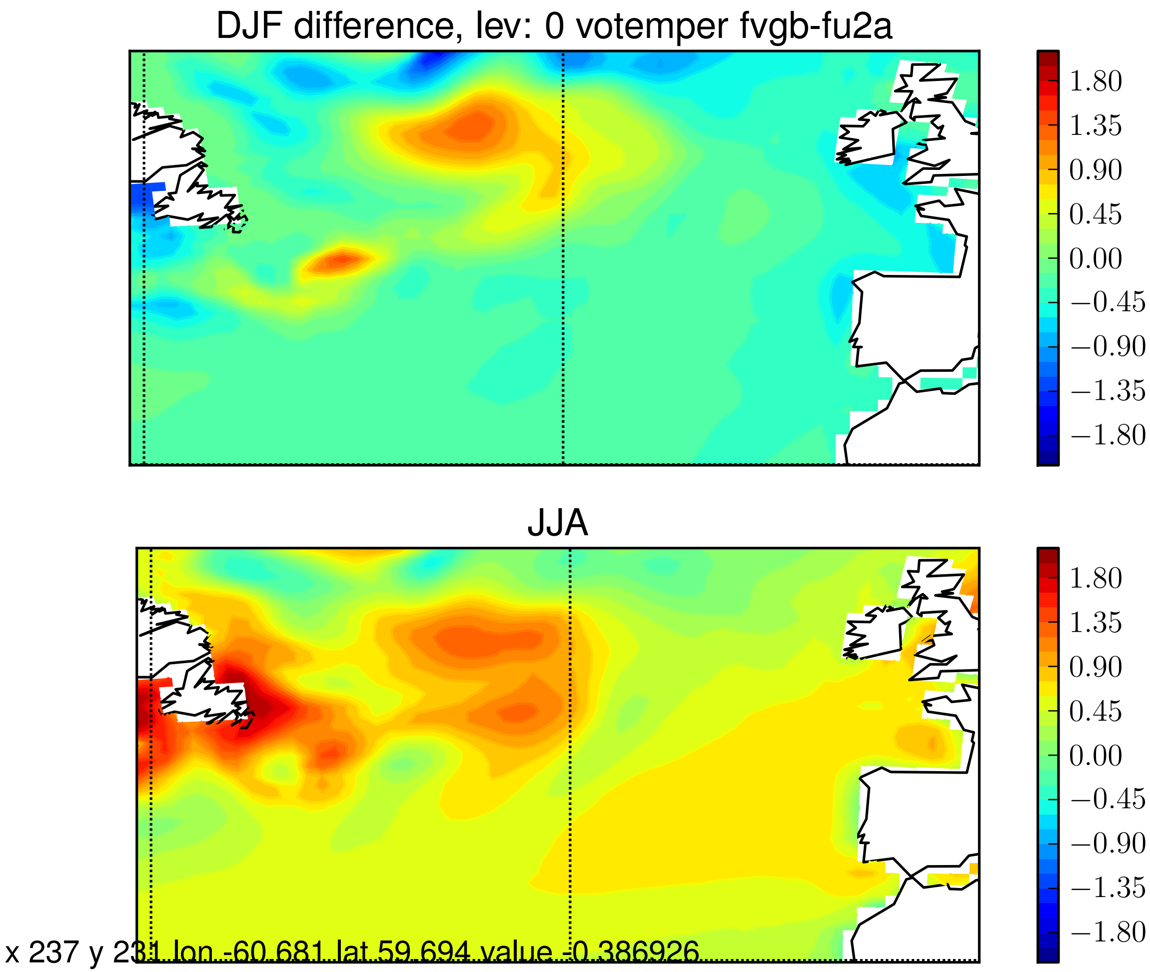

Figure 12: Combined impact on SST averaged over a 20 year period for the North Atlantic.

CONCLUSIONS

- The Stokes-Coriolis effect seems to have important consequences for the upper ocean circulations. Wave effects are felt down to the Ekman depth.
- By regarding air and water as one fluid it is evident that there is also a Stokes-Coriolis force in the atmosphere, but the importance of this for the atmosphere still needs to be assessed.
- The introduction of the surface drift enabled me to understand the ocean momentum budget in the presence of ocean waves. This drift is not only relevant because of theoretical considerations, but is also of practical value. According to me in a coupled atmosphere-ocean model the surface drift is a key player in the boundary condition for the atmosphere.
- Wave breaking enhances the upper ocean mixing and even affects the average SST field over a 20 year period.

Generalizing the expression for the surface drift

The result for the mean momentum of a single wave may be generalized to the case of many waves once the joint probability distribution function of amplitude and period of the waves is known. To this end we start from Eq. (25). This is an equation obtained for a single wave train. Without given the proof it is stated that the result (25) also holds approximately for a narrow-band wave train, i.e.

$$\eta = a(x,t)\cos\theta,$$

where a is a slowly varying amplitude, while also local wavenumber $k = \partial\theta/\partial x$ and local angular freguency $\omega = -\partial\theta/\partial t$ are slowly varying functions of space and time. From now onwards the envelope and local frequency are treated as random variables. In practice it is well-known that for linear wave trains the surface elevation η obeys a Gaussian distribution. As shown in the Appendix it is then straightforward to obtain the joint pdf of envelope a and period $\tau = 2\pi/\omega$. The ensemble averaged wave momentum then becomes

$$\langle \mathbf{P}_w \rangle = \frac{1}{2}(\rho_a + \rho_w)\int_0^z \mathrm{d}a \int_0^\infty \mathrm{d}\tau\ p(a,\tau)\ a^2\omega\ d(z,a),$$

where the joint pdf $p(a,\tau)$ follows from Eq. (A6). The relevant integrals may be evaluated and the following simple result is obtained:

$$\langle \mathbf{P}_w \rangle = \rho \langle \mathbf{u}_{surf} \rangle = \frac{(\rho_a + \rho_w)}{\sqrt{2\pi}} \, \sigma \overline{\omega} \, e^{-\frac{1}{2}(z/\sigma)^2}, \; \sigma = \frac{H_S}{4},$$

with H_S the significant wave height and with $\overline{\omega}$ the mean angular frequency equal to m_1/m_0 where m_1 and m_0 are moments of the spectrum. Hence, in contrast to the single wave case, for a spectrum of waves with random phase the profile of the wave-induced momentum $\mathbf{P}_w$ is a well-behaved function of depth as it is a Gaussian with width determined by the significant wave height.